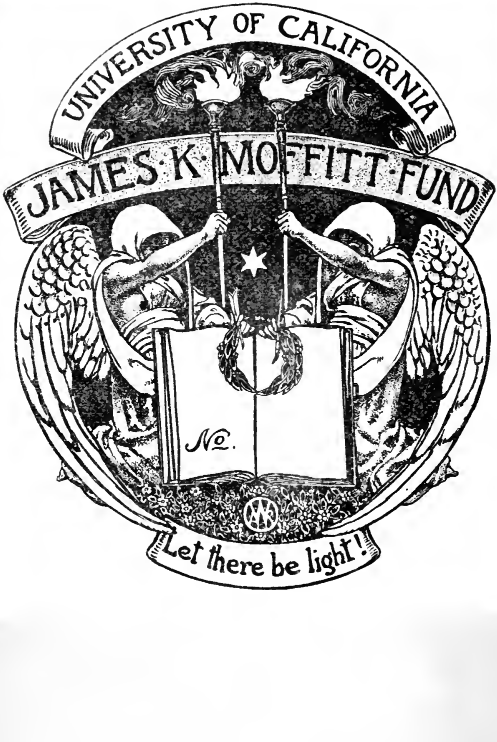
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No.
Let there be light!

AN

INTRODUCTION

TO

LOGICAL SCIENCE:

BEING A REPRINT OF THE ARTICLE "LOGIC" FROM THE EIGHTH EDITION OF THE ENCYCLOPÆDIA BRITANNICA.

BY

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PREFACE.

The reprinting of this summary will, at least, make it available to myself for the instruction of my pupils. Consideration, however, of the place which it was in the first instance to occupy, has impressed on it many features that would have been wanting in a mere text-book for lectures.

The nature of the topic, it is true, barred all pretensions to the construction of anything like a popular essay; and the position which it was thought right to take up, in relation both to the character of the science and to its treatment, had to be fortified by so much both of theoretical discussion and of technical interpretation, as to leave but narrow space for illustration, by examples or otherwise. But the original destination of the treatise did impose two duties: that of securing its usefulness to certain readers; that of doing one's best to make it deserve attention from certain others. I am willing to believe that there are reasons why it may be offered, in this separate shape, to persons of both classes. To those who are uninstructed in Logic, careful study of it will convey an adequate knowledge of the common rules and nomenclature. Adepts, again, who interest themselves in such inquiries speculatively and critically, may find that there are here presented some contributions to the exact history of logical science, and perhaps also some hints, not altogether trite, that tend towards the elucidation of its contents.

For any who may open the book as learners, a warning had better be premised. It will not be so easily mastered as are those English works of the sort which are most frequently studied. To myself, it must be owned, this does not appear

to be a fault. Logic, the easiest of all sciences, ought not to accept any assistance that would cost the very smallest sacrifice of its scientific character; and such sacrifice accompanies every turning aside from difficulties like those with which its students are here invited to grapple.

Of what kind the barriers are which are thus thrown across the road of the beginner, the initiated will perceive readily from explanations now to be addressed specially to them.

In the way of introduction it should be noted, that the design is limited to a development of the principles of Pure Logic, the laws by which thought is governed formally and universally. The uses to which thinking may be put, and the subordinating of which to the formal laws is or should be the function undertaken by codes of Applied Logic, are not touched on, unless when they prompt incidental illustrations.

Within our own province we shall come continually on ground, where we have only to tread contentedly in the footprints left behind them by the established guides. It is not, however, along those smooth and well-frequented paths, that the main line of the journey runs. In regard, indeed, to the validity of any of the received logical rules, there is as little room for controversy, as there is in regard to the truth of any of the proved geometrical propositions. But in the manner of theorizing the rules there prevail very remarkable diversities. A large majority of our English logicians have not held it necessary to dig for any foundation, deeper or broader than that which is laid by isolated and general appeals to common-sense. A minority, co-operating with foreign analysts, aspire to finding, in Logic, not a mechanical aggregate of technical rules, but the philosophical unity of an organic system of principles. Under the leadership of these speculative allies I respectfully volunteer to serve.

Two or three sentences will suffice for demonstrating the skeleton of the theory which it has been my aim to expound; and the outline may clear the way a little for those whose familiarity with Logic lies wholly on the practical side.—At the root of the science is placed, explicitly, *The Principle of Consistency or Non-Contradiction*, yielding the logical axioms of identity, difference, and determination. When this law is

developed with reference to the only modes of thought that demand to be exhaustively systematized, we gain a group of corollaries, the central point of which is found in *The Law of the mutual relation between the Extension and the Comprehension of Concepts and Common Terms.* The primary law having been evolved into this secondary law, the theory both of predication and of inference has virtually been reached. The complex and derivative law of the concept has been justified by its dependence on the wider and simpler law of consistency; and the formidable array of logical rules and processes, not only cumbrous, but confused, so long as its parts are contemplated separately, disposes itself into a symmetrical whole, when the law of the concept is accepted as the combining truth.—In the Introduction and the First Part, the purpose is that of setting forth the character and relations of those two laws, and fitting them for use as logical re-agents: in the Second and Third Parts, while the current rules of predication and inference are laid down and explained, their reasons are sought in the secondary law, and through it in the primary.

Both in the framing of the design, and in its execution, obligations have been incurred to many logical writers; and heavy ones to contemporaries, in this country as well as on the continent. All authorities, relied on for anything except what may be regarded as public property, are acknowledged in marginal references, which are meant to be fair and full. But allusion must be made even here, and could not well be made too early or too prominently, to one profound philosopher and scholar, whose services to Logic have been, though necessarily less celebrated, yet not at all less valuable, than those through which he has founded a powerful school in psychology and metaphysics. That which I, like others, owe to the few writings published by Sir William Hamilton, both for suggestions as to the principles of the science and for information as to its history, stretches very far beyond and around the salient points to be immediately indicated.

By a few recent logicians among ourselves, the Primary Law of thought has been apprehended very clearly. But the apparatus for treating it with precision must be chiefly bor-

rowed from abroad. Both the exposition of its character and conditions, and the subsequent tracing of derivative doctrines to this source, are here elaborated with a fulness which, by those who attach more importance to the *whither* of a rule than to its *whence*, will certainly be pronounced excessive.

The uses which are made of the Secondary Law produce a dissimilarity, not less decided, between this outline and our most popular books.

This law has long been adopted by the German logicians, as a basis for the method of analysis which they bring to bear, not on simple predication only, but also on division and definition. But even on those sections of the science there are reflected, from the speculations of the great thinker who has been named, lights which illuminate more dark corners than one.—Of the attempts now made to think out, by steps short and obvious, yet not seeming to have been distinctly anticipated, the applications (other than syllogistic) of the principle of the concept, there are two for which it may be allowable to solicit particular scrutiny. The negation of co-ordinates, while it is afterwards used for the dissection of the syllogism, is in the first place introduced, explicitly and emphatically, into the theory of definition. The conversion of propositions, an operation whose genuine character must rule very wide issues as to the syllogistic figures, is resolved into a transference of predication from extension into comprehension, or from the latter into the former.

To Sir W. Hamilton belongs, exclusively, the application of the correlation between extension and comprehension to the formal theory of the syllogism. Although hitherto little studied, it is the great achievement of his logical system. When all the propositions constituting any syllogism of the received scheme have been analysed with this reference, syllogistic reasoning unfolds itself under relations at once interesting and unexpected.—In this stage of the investigation it has been my endeavour, not simply to report, but to apply and extend systematically, the original researches which lay before me; and, accordingly, while the adoption of the new test raises large questions with which few students of the science are likely to be familiar, the details of the analysis

exhibit some views which, so far as my reading has informed me, had not previously been gathered from the premises. Whether those premises have been inferred from either clearly or conclusively, it is for others to judge. The fragmentary notices, which are still our only logical relics of the departed master, induce a belief that he would have condemned more than one of the consequences which, combining his data with others, I have ventured to deduce.

It should be said, further, that (with a reluctance dictating extreme minuteness in the assignment of reasons) I have felt myself compelled to abstain from admitting all those additional forms of assertion, the incorporation of which with the orthodox scheme makes up Sir W. Hamilton's "thorough-going quantification of the predicate." Of his four new propositional forms, there are positively adopted no more than two; both of which have also been marked, and very widely applied, by another logician of our day. Nor is it otherwise than as instruments towards certain ends, that even these are here used. The one is required, but is sufficient, for giving formal completeness to the theory of conversion, and to that of the ordinary syllogistic moods: the other completes similarly the theories of definition, of division, and of a kindred process, the perfect induction.—On the other hand, in all cases of nicety, obedience is thankfully paid to Sir W. Hamilton's singularly fruitful postulate, the express signature of quantity for the predicate.

In a word, those who are most extensively conversant with the modern phases of logical speculation, will be more inclined than others to believe, both in the possibility of presenting some of the most venerable doctrines in novel aspects, and in the desirableness of determining more closely than of old several problems, exhaustive solutions of which are indispensable to the ideal perfection of the science. The preceding sketch shows, generally, in what direction, and under what guidance, the region is now explored. In spite of all shortcomings, whether in method or in result, I do presume to hope that it will have been in my power to offer aids for instructive reflection, not, indeed, to the lovers of easy thinking, but to students armed by patience as well as sagacity for conduct-

ing complex processes of exact analysis. The treatise is far, also, from being a mere compilation. My task has by no means been confined everywhere to the arrangement, far less to the collection only, of materials that were already piled up openly round the mouth of the mine. Not a little that is here distinctively characteristic has been yielded by deposits which, though stronger hands had extricated them from the imbedding strata, lay as yet in deep and distant levels, and could not be raised to the surface without more or less of independent exertion. It is even the fact that a good deal of excavation has been performed (perhaps without disengaging much marketable ore) in some of the galleries that branch off from the shaft; galleries, too, in which the lamp still burns but feebly, and which have not been worked out to the end of the vein by those who opened them.

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LOGIC.

INTRODUCTION.

CHAPTER I.

The Psychological Data of Logic.

The relation between logic and psychology.

1. Logic is the theory of inference. Round this assertion circulate all endeavours towards precise definition of the science. Its function would thus be very incompletely described, if we were to refuse including, under the name of inference, any processes of thought having data narrower than those of the Syllogism. But we ought to comprehend, within the sphere of inference, all processes wherein a truth, involved in a thought or thoughts given as antecedent, is evolved in a thought which is found as consequent. On this understanding of the term, the Laws of Inference may rightly be said to be those which it is the function of Logic to develope into a system.

Logic, as being thus a systematic development of certain mental laws, takes its place among the sciences constituting the Philosophy of Mind. It is one of those derivative sciences, which branch off on all sides from Psychology, the one original and central science of the cycle.

The data which it has imperatively to demand from psychology are, doubtless, both fewer and nearer to the surface, than those which are required by any other science standing in the same predicament. They are so few, and so intimately related to each other, that in one page of the great psychological volume we read them all: they are so simple and obvious, that psychological controversies raise questions only as to the way of naming them, and leave the facts themselves quite untouched. They might be, and very often are, taken for granted without reference to the science which is their real source; and the borrowing is still further disguised when they are merely, one after another, brought to light as they are needed for use.

But there are more reasons than one for treating them differently.

The chief reason is this. The laws of thought which logic developes are necessary and universal. Therefore we are, especially if we aim at studying the science with scientific precision, in danger of forgetting that its truths, though not measured by experience, become known to us only through experience, that is, in the actual exercise of thought; that those forms of thought, to which all logical laws are relative, are themselves actually conditioned, and conditioned from without as well as from within; and that, if the laws are to have practical applicability as regulative canons of knowledge, their foundation must be firmly laid among the actualities of mental manifestation. The dependence of logic on psychology must be broadly asserted, in the way of protest against systems which seek to divorce it from experience.

Again, the province of logic cannot be clearly distinguished, unless its data have been expressly separated from

the uses to which it puts them. It must assume Apprehension and Judgment, as the Forms of those thoughts which are the constitutive factors of inference: it must assume the Laws, both subjective and objective, by which Apprehension and Judgment are universally governed. Its duty is the development of those laws as bearing on those forms.

Lastly, however readily the data of our science might be admitted if presented in a loose and unscientific shape, it is by psychology that they have been systematized, justified, and designated. They are not available for precise and exhaustive use, unless they are laid down and named with the utmost exactness which psychological analysis has made it possible to attain.

The modes of consciousness not cognizable by logic.

2. It may be desirable to begin our hasty psychological survey, by explicitly setting aside those classes of mental phenomena whose laws are not logical.

When the phenomena of consciousness are considered subjectively, or purely as functions of the conscious mind, they seem to be naturally distributable into four Primary Modes. With three of these logic is in no way concerned. It does not deal with Feeling (or cognition without distinct evolution of the objects), in either of its objective varieties of sensation and emotion; nor with Wishing or Appetency, either as desire or as aversion; nor with Volition, the consequent of wishing, as that is of cognition. Its sphere lies wholly within the fourth of the modes; that is, among the facts which are describable as Thinking or Cognition, pure and proper: in other words, it lies among the phenomena which only are strictly describable, in the current phrase, as operations of Intellect or Understanding.

Nor is it as to all of these that logic requires to assume

anything. It ignores, especially, one of the two great divisions into which, through differences in the character of cognizable objects, all human thought or knowledge is distributable. We know or think of an object, either directly, or through another object which represents it. Knowledge of the former kind has been called Immediate, Intuitive, or Presentative; that of the latter kind, Mediate or Representative. All objects that are cognizable immediately, may also be known mediately or representatively; but by far the most valuable part of our knowledge has objects which are cognizable mediately and not otherwise.

It is only with facts of mediate knowledge that logic can deal. It is when, and only when, reproduced from the past in present facts of thought, that immediate cognitions or their objects are susceptible of analysis or evolution. Consequently, these yield materials open to logical scrutiny, when, but only when, they are so reproduced; while, further, there is exposed to such scrutiny the whole gigantic mass of those complex cognitions, whose prominent elements are objects mediately known, and in which immediate cognition supplies only, as it always must, elements which are implicitly and obscurely assumed.

Discursive thought the matter of logic.

3. From the field of logic there are thus shut out all those mental facts, which are not contained in the sphere of mediate thinking. Within that sphere, the field of the science receives still another limitation. The only matter with which it deals is that which the schoolmen called Discourse or Discursive Thought.

The name hints at the character of the thing. Discursive thinking is a passing from thought to thought. Logic evolves, not laws which govern any one fact of mediate

thinking taken singly, but relations between two or more such facts, or laws which govern the derivation of one such fact from another or others. That which logic scrutinizes is not one fact of thought, but a process constituted by a plurality of such facts. It considers Thinking as Knowledge or Cognition, that is, as having objects which are truths; but it assumes and systematizes those laws only in virtue of which, one or more facts of knowledge being given, other facts of knowledge may be elicited from them. The logical question is not, whether a given judgment or assertion is true or false. It is only whether, in virtue of certain laws of thought, there does or does not subsist, between two or more judgments or assertions, the correlation of antecedent and consequent; whether, one or more of the assertions being admitted, another must be admitted, or must be denied, or may be either denied or admitted. In short, the processes whose laws the science digests, possess always the essential characteristics of inference; and they are always, also, reducible into a form to which that name is directly and properly applicable.

Psychologically or subjectively considered, discursive thought exhibits no distinctive characteristics beyond those which belong to it as being necessarily mediate or representative. It is always resolvable into a series of judgments. Its peculiarity lies in the relation between the constitutive judgments: it is a relation in which the objective side is the more prominent of the two. We might say, indeed, that the relation subsists, not between the acts of judging, but between the judgments; not between one mental fact and another, but between their several results or products. The ideas which are the factors of each judgment must represent objects which, if not real, are at least thinkable: each judg-

ment is given to logic in that aspect. It is only after having been so viewed, that the judgment is, as it were, turned round, to be examined from the opposite, the subjective side. A certain relation between thinkable objects being assumed in the antecedent judgment or judgments, the Laws of Thought compel us to think another relation between thinkable objects in the judgment which is the consequent.[1]

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[1] In the nomenclature of the German schools, the name Thinking or Thought is confined (at widest) to Thought Discursive. The same limitation has of late come into use among us. It is adopted by Sir William Hamilton, who acknowledges only this meaning of the word, and that other in which it covers all kinds of mental phenomena. "Thought and Thinking are used in a more and in a less restricted signification. In the former meaning, they are limited to the discursive energies alone; in the latter, they are co-extensive with consciousness." (*Edition of Reid*, p. 222.) In this view, perception, whether external or internal, is not thinking in the narrow sense of the term; neither is imagination, whether it be simply reproductive, on the one hand, or creative or synthetic on the other.

In the text, the word Thinking is used as synonymous with Intellect or Intelligence. In the psychological scheme which was hinted at in the last section, Thinking, Intelligence, Cognition, is regarded as distributable into modes or kinds on each of two principles. Considered subjectively or formally, it must take place in the one or the other of the two forms which are called Apprehension and Judgment. Considered objectively, that is, as modified by the character of the objects known or thought of, thinking falls, first, into the two genera of Immediate and Mediate. Immediate thinking is of two species, Self-Consciousness and Consciousness Perceptive (perception, internal and external); facts of both kinds being indeed actually complex, and especially having Feeling as an element, but both being susceptible of being regarded abstractively as facts of pure and proper thinking or cognition. Mediate thinking embraces two species; first, Imagination proper, that is, the think-

4. All the laws of discursive thought bear on certain Forms, in which, and in which only, the facts of mediate thinking which constitute the process are possible.

Mediate thinking formally distinguishable as apprehension or judgment.

Mediate thinking must always take the one or the other of two forms; forms whose difference is as truly subjective or formal as those which mark the four primary modes of consciousness. It must be either a fact of Apprehension or a fact of Judgment; species recognised most readily through the test of expression as brought to light by logical analysis. Every fact of thought expressible by a Term (that is, by a word or words interpretable as the name of an object or objects), is formally a fact of apprehension. Every fact of thought expressible by a Proposition or Assertion or Predication, is formally a fact of judgment. Every thinkable object, or group of objects, no matter how complex our thought of it may be, is denotable by a term, if only we have words adequate to express all the elements which we think of it as involving. Every act of thought in which we explicate a relation, is denotable by a proposition, and requires the propositional form.

5. (1.) The first formal determination of Judgment is one which, as we shall find, lies at the very root of all logical doctrine. A judgment, or the proposition which expresses it, must always be either Affirmative or Negative: a

The formal characteristics of judgment.

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ing of individual objects not present (which, again, may be either simply reproductive or synthetic); and, secondly, Conception proper, or the thinking of universals. Both Imagination and Conception take spontaneously the form of Apprehension; but both, besides presupposing Judgments, yield matter for new Judgments, which are necessarily Mediate.

judgment which should be neither the one nor the other is utterly inconceivable: there is no medium between affirmation and negation.

(2.) Every judgment, further, is formally resolvable into the affirmation or denial of a Relation; and relation implies plurality of ideas or objects related. We shall immediately, it is true, encounter a class of cases in which there is not really such a plurality; and the possibility of these will at once prescribe a limit to logical analysis, and serve as a point of departure for the development of judgments really founded on relation. In the mean time, it must be noted that the formally relative character of all judgments, impressing itself necessarily on the propositions by which all judgments must be expressed, makes it possible, while for exact logical scrutiny it is necessary, to dissect all propositions into three factors or constitutive elements. They are these: the two Terms (Subject and Predicate), which are names of the ideas or objects correlated; and the Copula, in which the relation is asserted. The subject denotes that which is the datum or antecedent of the judgment, that which is given to be determined by the other term. The predicate denotes that which is the quæsitum or consequent, that by which the subject is determined. The copula asserts the relation, but it asserts nothing more; and, that we may make the closest possible approach to a pure affirmation or denial, it must always, for strict logical use, be either "is" or "is not," "are" or "are not." It might be said, that the terms are the objective factors of a proposition, and that the copula is its subjective factor.

(3.) There emerges thus, as necessary to be assumed in all further steps, the doctrine of that which logicians call the *Quality* of propositions. Judgments and propositions

must be either *Affirmative* or *Negative*; and the quality of a given proposition is signified by its copula.

6. The formal theory of judgment, in itself exceedingly simple, becomes perplexingly complicated through the complexity inseparable from the theory of apprehension.

The formal characteristics of apprehension.

Apprehension, as the name is here understood, is the Simple Apprehension of the logicians; that is, mere apprehension not evolved into judgment. We give the fact that name, when we desire to describe it by reference to the thinking subject, or as a mental act or phenomenon: when we desire to describe it as representative of an object or objects, it may be, and is, called an Idea or Notion. The idea or notion is that which is directly denoted in language by a Term; and a term is thus, mediately, the name of an object or objects. The names apprehension and idea denote one and the same fact; but they denote it as regarded from two opposite points of view, from either of which it may be contemplated, but not from both at once.

The differences between objects apprehensible, must modify variously the character of the ideas and terms through which they are thought. But, of all such differences, there is only one which modifies the form of apprehension necessarily and always, and which, therefore, possesses a peremptory logical value. It is the difference between the Individual and the Universal. This difference yields two varieties of apprehension; namely, Imagination and Conception. We apprehend the individual in imagination, which, objectively viewed, gives an Image: we apprehend the universal in conception, which, objectively viewed, gives a Concept. The image is expressed in words by a Singular Term; the concept by a Common or Generic Term.

The Singular Term is a name for an object thought as having unity, or as being one object. Its unity or individuality may be constituted by parts, each of which might in its turn be thought as one; but it is thought under some relation yielding a unity, which cannot be thought away until some other relation is substituted for the first. "Aristotle," "John Milton," "This man," are not more distinctly singular terms, than are these: "The course of conduct to be adopted," "The (individual) series of fancies which lately floated through my mind." "Yonder forest" is a singular term; so are "That tree of yonder forest," "The gnarled bough of that tree."

The Common Term is the Name of a Class, a name for a plurality of objects, a name applicable to any or all of them in respect of a certain relation between them. Its meaning as a name of objects is not exhausted unless it is applied to all; as "All poets," "All the trees in the wood:" but, continuing to think of the objects under the same relation, we may apply it to some, or to any number of objects fewer than all, as "Some poets," "Most of the trees in the wood."

One feature of contrast should here be noted, as having a wide logical applicability. Apprehension may be either Direct or Symbolic. Imagination is a direct apprehension: if we think of an individual object through a name, we think symbolically; but it is not necessary we should so think of such an object. A person remembered is thought of directly when we call up his image, the representation of his appearance. Contrariwise (and this is the point to be noted), conception is necessarily symbolic. That which is signified by a common term, cannot be represented in thought otherwise than through a symbol; and words, if not the only possible symbols, are the only ones that are fully adequate

for the purpose. Why the case should so stand, is a question very abstruse, and not logical. But some of the reasons may come to light when we have examined the common term a little more closely, and have discovered that it represents, not anything actually known before, but a complex thought which has resulted from a comparison of known objects. In the meantime it should be remembered that, in a certain view, "concept" and "common term" mean one and the same thing; that, at the very least, the thought which we call a concept is not only not expressible, but not even thinkable, unless through the common term.

The extension and comprehension of common terms.

7. The signification of the common term is double. It is a name both of substance and of attribute: it is a name both of objects possessing an attribute, and of an attribute possessed by objects.

Most obviously it is, as it was already described, the name of a class, of a plurality of objects. But it is a name of these as thought under a relation; and that relation is, their possession of a common attribute. It is, indeed, applicable to all and each of the objects, just because, and in so far as, they are thought as possessing a certain attribute, or a combination of attributes, which combination is usually thinkable as one attribute more or less complex.

If we attempt to trace hypothetically the formation of a common term, we shall find that the discovery of the attribute must have preceded the imposition of the name.

One of the conditions under which only we can think of objects, is that of Number, which developes itself in the phases of unity, plurality, and totality. If our given objects are more than one, our contemplation of them is obscure and unsatisfactory: we constantly strive in thought

to attain unity. But, an individual and simple unity being here unattainable, we endeavour at least to gain a complex unity, that is, a totality constituted by parts. We endeavour to think of our plurality of objects in a relation in which they are such constitutive parts. But such a relation must be that of resemblance: it must lie in the fact, ascertained by us, or for us, through observation, that each of the objects possesses a certain attribute or property. The common term, borrowed or invented, will then enable us to think of our objects as being "all," but only as being "all" in respect of their possession of the common attribute.

Now, this two-fold relation of the common term is the most fruitful of all logical data. Therefore we had better seize, at once, names by which both of its members may be technically described. The objective relation of the term, its signification as being a name of objects, will be called its *Extension:* its attributive relation, its signification as a name of attribute, will be called its *Comprehension.* The common term "man" has extension, as being a name for al and each of the persons constituting the class; it has comprehension, as being a name for the attribute "human nature."

The Extension of the common term is, naturally and necessarily, the more prominent relation of the two, in thought as well as in expression. It costs an effort to think, and it requires an abstract form to express, the common term as the name of an attribute. The common term, as the name of a class of objects, is readily thought, and finds its expression in a concrete form. Again, the relation of number, or of whole and part (involving quantity in one phase or another), is an element of every thought in which a concept is one of the factors. The question must always be

raised, whether the objects thought of are all, or only some, of the objects constituting the class: the question is, in other words, whether a given term is, in a case under examination, used in the whole, or only in a part, of its extension.

There emerges thus the doctrine of that which logicians call *Quantity*. Every common term must be considered with reference to its quantity. It must either be *Distributed*, that is, applied to all the objects of the class; or it must be *Undistributed*, that is, applied to fewer than all of the objects. Distribution and non-distribution are indicated by prefixed Quantitative Signs: "all" or "any" for the former, "some" for the latter. A proposition, again, is specially said to be *Universal* when its subject is distributed, that is, when the antecedent of the judgment is the whole of a class; it is said to be *Particular* when the subject is undistributed.

Generalization and specification.

8. Logic does not require to assume that common terms have undergone any deeper probing, than that which detects in them the formal expression of the correlation between substance and attribute, and of that reference of objects to classes for the sake of which the correlation is thought of. But both their character, and the limits which circumscribe logical dealing with them, may be more clearly understood, through a cursory glance at those objective conditions by which their formation is determined.

Classification, as a process yielding concepts and terms which import real knowledge, is very far from being arbitrary. When we endeavour to refer objects to a class, in virtue of a common attribute, that which is sought is, in effect, some one law under which we may know or believe

that all the objects are placed. But every thinkable object is, in virtue of the complications involved in life and nature, amenable to many laws, and may therefore be thought of as possessing each of many attributes. Each individual object may be placed in any of many classes, or have affirmed of it any of many common terms, denoting attributes common to it and to other objects, that is, laws which both they and it obey. So, likewise, of any class of objects (if we set aside the unpractical case of a class wide enough to contain all others), it must be affirmable that it is included in some other class; while of most classes it must be affirmable that they are, when considered with reference to diverse attributes or laws, included in each of many others. Thus common terms are affirmable of each other; and it is out of such affirmation, with the negations accompanying it, that there comes the only matter of reasoning difficult enough to reward scientific scrutiny, or complex enough to bring into play the highest logical doctrines.

Again, the classes which are thus comparable can very seldom be co-extensive: those which we do compare in ordinary facts of thinking never are so. Common terms, accordingly, distribute themselves into systems, each of which constitutes a graduated series. A class containing certain objects is placed in a class containing these objects, together with others. This second class is similarly placed in another, containing all its objects, but not these alone; and the series may so rise in many successive steps. Thus we may pass from "man" to "animal," from "animal" to "creature organized," and thence, if we will, to "created being." When terms are taken in such an order, their extension increases at every step; each succeeding class is thought of as containing more objects than the class which last preceded it.

A very little reflection will show that, contrariwise, the comprehension of the terms has decreased at every step. Each succeeding term implies an attribute (simple or complex) fewer than that by which it was last preceded. "Animal," being the name of a class containing objects besides "man," ceases to suggest the attributes which distinguish man from those other objects; and "organized creature," as being the name of a class containing both "animals" and "plants," ceases to suggest the attributes which distinguish "animals" on the one side, and "plants" on the other. In short, when we think according to this order, we are, step by step, thinking *in* objects, and thinking *out* attributes. This is the course of thought which is usually called Generalization.

The counterpart of it is the process of Specification or Determination. In it we begin with the most extensive class, and descend, step by step, till we reach the lowest. In so doing, we are, quite as evidently, thinking *out* objects and thinking *in* attributes. Each successive class in the descent contains fewer objects than the last; but each possesses, in addition to the attribute of the preceding class, the attribute possessed by its objects, and wanting to the objects which with it make up the class preceding.

Thus there comes to the surface one of the most valuable of all the laws from which logic draws corollaries bearing on inference. It is the *Law of the Inverse Ratio* which subsists between the Extension of common terms and their Comprehension.

Corollaries as to common terms.

9. There may be noted, further, in the way of corollaries, one or two features of the common term, which are important as bearing on its logical uses.

(1.) The concept tends to fall back into the image. It

is usually held that all class-names must originally have been the names of individuals; nor is it easy to suppose any other source for them. At all events, when we consider the existing state of language, without speculating as to its formation, it becomes evident that the common term, as being a name applicable to every object of the class, has a suggestive force, leading us downwards towards imagination of objects individual. In this aspect, the term may be regarded as giving an inadequate idea, an obscure and vague image, of the individual,—an image that becomes more and more indistinct, the wider the generalization is which the term presupposes. Concepts, therefore, though properly representing, not objects, but the manner in which we think of them, cannot entirely lose their hold of objects; and reality, actual existence, operates as a normal limit to the formation of concepts and common terms.

(2.) The concept, however, *qua* concept, is not used for the purpose of suggesting any reference to individuality. It is a thought of relation, and therefore a complex thought: its elements are discoverable, and may be brought to light in the form of a judgment. The signification of a common term is most clearly and fully perceived, when it is regarded as being an abbreviated symbol of a complex proposition, the import of which might be formulized in some such shape as this: "All the objects thought of are objects possessing a certain attribute, and therefore constituting a certain class."

Indeed, this manner of considering terms, as being short-hand expressions for propositions previously gained, is capable of being put to very various uses. All logical rules which are easily available, and all primary logical principles, are brought to bear by the separate extrication, from pro-

positions, of the terms which are their objective factors. If we attempt logically to compare propositions without such dissection, they can be treated only through the cumbrous and derivative rules of hypotheticals, or other complex forms of predication. Categorical predication, the normal and simple expression of judgment, yields the terms immediately and easily. But such predication is often not attainable till propositions have been condensed into the form of terms; and it is, perhaps, traceable always to such a condensation, which we perform spontaneously and continually, guided by the irresistible desire of making language keep pace, as far as its natural slowness will allow, with the electric rapidity of unexpressed thought. This condensation comes into action with especial frequency in our thinking of universals; since these are never thought unless through words.

CHAPTER II.

The Function and Axioms of Logical Science.

The two-fold code of discursive thought.

10. Knowledge requires both to be gained and to be verified. It is desirable that we should obtain aid, through systematic laws, both for discovering new truths and for testing the results of alleged discovery. A Theory of Derivative Knowledge would be complete, if it issued a twofold code, ruling, with scientific accuracy, processes of both kinds.

A system aiming at the former of these ends is properly constructive or positive: if it is capable of justifying its promise, operations directed by it will yield positive additions to our knowledge. A system aiming at the other end is no more than regulative or negative: it can only enable us

to decide, whether that which is presented as knowledge deserves or does not deserve the name.

It is confessed by all, that the Code of Discovery has never yet been thoroughly digested; it is believed generally, and perhaps universally, that it must always at many points remain imperfect. In all the shapes in which it has been promulgated, it is described commonly, though neither quite correctly nor quite completely, as the Philosophy of Induction. Some of those who have legislated for this region of logical science maintain it to be practically independent of the other; not that they hold the testing of results to be unimportant, but that they believe this duty to need no scientific assistance, and to be safely left to native sense and practised sagacity. By such thinkers, the laws of discovery are asserted to constitute the only logical system that is worthy of study. Others allow, more correctly, that a developed theory of the processes by which thought may be tested, is imperatively necessary as the foundation for a theory of discovery. These speculators usually consider the systematized theory of induction or discovery as constituting, in one department or in several, an Applied or Particular Logic; in respect that it is a scheme in which logical laws, the laws for the testing of thought, are applied to special uses, varied by the varying character of the purpose and the matter.

The Testing of Discursive Thought is the function undertaken by that system of logical science, which has been called the Aristotelian, from its founder or greatest expositor; the Syllogistic, from the process which is its highest development. It has been spoken of as a Pure Logic, because it is, or may be made, as far free from assumptions foreign to it, as any science can be which has human thought for its matter, and by which, therefore, certain laws of the human

mind must be taken for granted, on the faith either of ordinary experience or of psychological analysis. It has been called Universal Logic, because its laws are applicable indifferently to all processes of discursive thinking, whatever may be the kind of the matter or objects thought of. Often, also, for the last of these reasons, it is called Formal Logic: its laws are laws, not of the matter of thought, but of the form or manner in which the matter is thought of. It professes to assign laws through which, on the assumption that the data of derivative knowledge are true, it may always be determined, either that the results are true, or that their truth or falsehood is not fixed by the data. This profession the science makes good, with a comprehensive precision which has, paradoxically enough, been turned into a ground of objection to it.

11. That which will here be attempted is an exposition of the laws constituting the science of Pure or Universal Logic. These, indeed, are the only laws which can correctly be called logical. The theory of discovery is logical so far only as it rests on those laws, as it must do by implication even when it does not expressly assume them; and it is only through them that the process of discovery can be philosophically theorized, with reference either to its capabilities or to its shortcomings.

First determination of the function of logic; the regulation of explicative thought.

It is difficult, perhaps impossible, to reach a formal definition of Logic, which shall at once mark out precisely the limits of the science, and describe its function clearly and exactly.[1] All the purposes of such a definition will be at-

[1] In the following definitions and illustrations, that which is signified by the name "Thought" is discursive thought. "Logic is

tained, if we can apprehend correctly these three points: first, the character of the mental process which the science examines; secondly, the character of the law which regulates the process, and the development of which is the duty undertaken by the science; thirdly, the character of those objective conditions under which only the process is possible, and with reference to which, therefore, the law must be expounded.

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the *à priori* science of the necessary laws of thought, with reference, not to particular objects, but to all objects whatever." (Kant, *Logik; Einleitung*). "Logic is the science of the rules of thought." . . . . "It takes no account of differences among the objects. It contains, therefore, rules for thought as thought; and these rules must consequently be universal and necessary, that is, they must be laws." (Kiesewetter, *Logik*, i., pp. (5) 7, ed. 1824). "Pure logic is the science of the form of thought." (Hoffbauer, *Logik*, p. 27, ed. 1810). "Logic is *the science of the laws of thought as thought*—that is, of the necessary conditions to which thought, considered in itself, is subject. This is technically called its *Form*. Logic, therefore, supposes an abstraction from all consideration of the *matter* of thought—that is, the infinitude of determinate objects in relation to one or other of which it is actually manifested." (Sir W. Hamilton, *Edition of Reid*, p. 698). "Logic is a formal science: it takes no consideration of real existence or of its relations, but is occupied solely about that existence and those relations which arise through, and are regulated by, the conditions of thought itself." . . . . "Logic is discriminated from psychology, metaphysics, &c., as a rational, not a real—as a formal, not a material—science." . . . "It has, in propriety of speech, nothing to do with the process or operation, but is conversant only with its laws." (Hamilton, *Discussions*, pp. 144, 136, 134). "Analytical logic is the science of the formal laws of inference." (Karslake, *Aids to the Study of Logic*, part i., p. 11). "Logic is the science of the laws and products of pure or formal thinking." (Mansel, *Prolegomena Logica*, p. 245).

(1.) Logic is the Regulative Theory of Explicative Thought. If it were necessary to lay down an express definition of the science, this assertion might be offered as being such: all closer examination yields only explanations of it. But not a little explanation is required.

Thinking may be either Explicative or Analytic on the one hand, or Ampliative or Synthetic on the other.[1] For processes of either kind, there must be given one thought at least—a datum or Antecedent. We explicate that thought when we extricate or evolve from it another thought, a thought describable as a Consequent of the first. Inference, reasoning, discursive thought, is merely explication of thought through analysis. If, at any step in the progress of our thinking, we assume any thought not involved in those which had previously been given or evolved, we ampliate our thought, we augment the matter of our thinking by the addition of a new datum or antecedent. If this new datum is synthesized or combined with our old ones, or with the thoughts which have been explicated from them, we may institute a new process of explication, in which we infer from our ampliated aggregate of data.

It is for pure explication only that logic is competent to legislate. A process to which logical canons are applicable, must be one in which there takes place nothing beyond this; that the constitutive elements of a given thought or thoughts are detected through analysis, and that there is evolved some thought which was involved or implied in the thought or thoughts given. The consequent differs from the antecedent in this only; that the former explicates, brings to

[1] Analytic and synthetic (Kant); Explicative and ampliative (Hamilton).

light, enables us to think distinctly, something which in the latter was only implied, and, therefore, thought more or less obscurely. Derivative thought, like water flowing through conduit-pipes, cannot rise above the level of its fountain. The truth or falsehood of the thought or thoughts assumed as the starting-point, must be determined by objective considerations, not by the laws of thought. In the same predicament is any uninvolved thought that may be interpolated in the course of the evolution; and, indeed, the introduction of any such thought just makes the beginning of a new process of evolution.

Second determination of the function of logic; the development of the principle of non-contradiction.

12. (2.) Explicative thought is regulated exclusively by one law, the Law of Consistency.

The character of this law determines, in several successive steps, the character of the process of explication.

In the first place, it determines the character of the antecedent. If there could be any such thing as a simple thought—a thought which is not analysable into constitutive thoughts, or in which no other thought is implied—such a thought would not be explicable or subject to the law of consistency; and if, in any individual case, we cannot discover what thoughts a given thought implies, that thought is for us inexplicable. Accordingly, a thought given for explication must be assumed to be complex.

Further, the complex thought, given for explication, must be resolvable into the thought of a relation. What this relation is, we cannot think clearly, unless in the form of a judgment, expressible by a proposition. When we ask whether ideas or terms are consistent or inconsistent with each other, the question really is, in what manner the relation presupposed between the ideas qualifies them for being combined

as terms of a judgment. Further still, the testing of consistency or inconsistency cannot be exhaustive, until the judgment has been analysed into the elements which in the proposition are signified by the subject, the predicate, and the copula.

Logical rules being most conveniently enounced with reference, not to the judgments and compared ideas, but to the propositions and terms through which these are expressed, the law is technically named with the same reference. It is thus called *The Law of Non-contradiction.*

This law, then, might be thus set forth, with reference both to the thought and to its expression, and in the negative aspect which is fittest for laws having uses regulative or prohibitory. "Ideas must not be combined in a judgment, in a form inconsistent with the relation presupposed between them or the objects they represent. Terms must not be combined in a proposition, in a form contradictory of their presupposed signification."

Third determination of the function of logic; the development of the principle under universal objective conditions.

13. (3.) This law, self-evident to the extreme of triviality, is not available for use until it has been specificated in more degrees than one, through consideration of the Character of those Objects which are Thinkable.

Thinking is possible only when there is given to it matter to be thought of: there must be not only a thinking subject, but a thinkable object. Thinking, accordingly, is conditioned, limited, determined, in each of its two opposite relations. It is conditioned not only subjectively, that is, by the laws which regulate thinking as a function of the thinking mind, but also objectively, that is, by the character of the objects of which it is possible for man to think.

Logic is enabled to elicit formal laws which are universally applicable, not by achieving the impossibility of ignor-

ing the objects of thought, but by considering, of the differences in the kinds of objects, those only which necessarily modify the form of thought or the manner of thinking. Now the number of those differences is the smallest that admits difference at all. We think of objects either, first, as having existence actual or possible; or, secondly, as having also mutual relation. The former of these objective conditions yields the idea of Individuality, the latter that of Universality. Under the one idea or the other all thinkable objects are thought. Individuality, as being the form of existence, lays the foundation of knowledge through intuition; universality, as being the form of relation, makes representative thought available as the instrument of knowledge explicative or discursive.[1]

ll propositions resolvable into assertions of identity or difference.

14. The law of non-contradiction receives its simplest application in Judgments whose Objects are Individual. This application likewise yields the normal form of the law, the form from which all other forms of it are derived, and into which all of them are in the last analysis reducible. That, in this its simplest shape, the law is (as we shall see) not so expressible as to avoid the double censure of triviality and barrenness, is a fact which would prove only, if proof were needed, that thinking which attempts to compare objects merely as individuals, without regard to their attributes or laws, not only requires no express rules, but is wasted on matter which can yield no real development of knowledge. It is nevertheless true, and demonstrable, that the most complex reasonings in which classes of objects are compared, owe their validity to that one self-evident prin-

[1] See Note at the end of the Chapter.

ciple, and that all logical canons are merely corollaries from it.

It is indeed impossible to think, even of individuals, unless under the condition of relation. Individuality implies the relation of number. Therefore, at the very outset, there comes up an indirect refutation of the possibility of constructing a system of logic, which shall be purely a theory of thought, and shall presuppose nothing whatever in regard to the objects thought of.

If, then, a thought having an individual object or objects is to be explicated into the form of judgment, there stands, as a barrier at the entrance of the field, that subjective law of thought which determines all judgments as being either *affirmative* or *negative*. We must either affirm or deny: no other form of judgment is possible.

Whether, again, we are to affirm or to deny, is a question determined by this law, which governs thought in its relation to all thinkable objects. "All objects are primarily thought of under the one or the other of the counter-relations of *Identity* and *Difference*." Of objects individual, when we attempt to consider them purely in their individual aspect, this is a palpable truism. Any one given object is identical with nothing but itself; it is non-identical with every other object. If, then, one object only is given, and if an affirmative assertion is demanded, the only such assertion which the case allows is the tautological and trivial affirmation,—"The object is itself: A is A;" which is an application of the equally barren formula,—"Every thing is that which it is." If, again, with the same datum, a negative assertion is required, we can frame only this negation, —"The object is not any thing which is not itself: A is not any thing which is not A (=A is not Not-A);"

or, in the formulized shape, "A thing is not that which it is not."

It is needless to say that, plainly, any other assertions than the two set down, would be contradictory of the assumption of the individuality of A.

But it must be asserted, broadly and peremptorily, that all the laws of inference are resolvable into this doctrine:—"An affirmation is an assertion of identity; a negation is an assertion of non-identity; and no medium is thinkable between the one assertion and the other." In laying down this proposition, we allege the Law of Non-contradiction, and couch it in a shape pointing straight to its place as the central law of logical science.

The three logical axioms yielded by the principle. The laws of identity, difference, and determination.

15. The law of non-contradiction is one and indivisible. But it may be regarded from any of more points of view than one; and one of these will suggest itself rather than the others, when the law comes to be applied on any special occasion. Accordingly it developes itself in one or another of three specific forms, which admit of being stated as three separate canons. These may be described as being *The Three Logical Axioms.*

Each of these, it must carefully be noted, is merely a partial evolution of the one central law; each of them implies the other two, and would lose not only its force, but even its meaning, if either of the others were wanting. It thus becomes extremely difficult to keep them separate in expression; and, besides this, it is often a matter of choice, to be fixed by the particular aspect in which we contemplate a given process of thought, which of the three is to be held as the rule directly bearing on it. For all these reasons it may be doubted, whether the older practice of

leaving the law unevolved, was better or worse than that attempt at distinct evolution of it, which has been adopted by the ablest of the modern logicians, and is here followed in deference to their opinion.

The logical axioms may be introduced in some such shape as the following:—

First, Affirmative judgments are ruled by the Law of Identity. "An affirmative proposition is not secured against inconsistency, unless its predicate may be thought as identical with the subject." The primary formula of affirmation is this:—"A is A."

Secondly, Negative judgments are ruled by the Law of Difference or Non-identity. "A negative proposition is not secured against inconsistency, unless its predicate may be thought as non-identical with the subject." The primary formula of negation is this:—"A is not Not-A."

Thirdly, Any two ideas must be either affirmable or deniable of each other. "Of any term as subject, any other term must be either affirmable or deniable as predicate." This axiom is usually called the Law of Excluded Middle, a name intimating the impossibility of any assertion intermediate between the two. It has also been called the Law of Determinability or Determination. The formula is this:—"Every thing is either A or Not-A: every thing is either a given thing, or something which is not that given thing."[1]

[1] The only point seeming to require comment is the position of the Third Axiom, which some logicians have mistaken so far as to attempt deducing it from the other two. It lies, in fact, subjectively deeper than either of them: a proposition disobeying either of them would be inconsistent with its data, but yet possible; a proposition disobeying the third axiom is inconceivable. The determination towards either affirmation or negation is a law of judg-

Even when viewed from this distant and somewhat hazy station, the axioms yield, more easily than it can be gained otherwise, one distinction which we shall find to be very widely useful. Two Terms, differing in this only, that the one wants, while the other has, the prefixed symbol of negation, are said to be Terms Contradictory. Thus, A and Not-A are contradictory terms. Two terms so related cannot be either affirmed or denied of the same object or group of objects. Of whatever object or objects A may be accepted as a name, Not-A must be understood as a name that covers every thinkable object besides. Such terms are the only terms which are formally and necessarily exclusive or contradictory of each other. If any two terms not formally so distinguished are held to be contradictories, it is because their relation is thought as being equivalent to that of formal contradiction.[2]

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ment, discoverable before all scrutiny of objective relations. Yet the third axiom, in the shape in which it has just been couched (or in any other making it available for use), is not independent of the other two. Though we know before-hand that, if we are to assert at all, we must either affirm or deny, we yet do not, surely, know what terms must be affirmable or deniable of what others, till we have, through the first and second axioms, resolved affirmation and negation into assertions of identity and difference. This resolution being made, we are reminded that any two terms must denote either one and the same object, or two objects which are different. If the former is the case, there is ground for affirmation; if the latter, there is ground for negation: and thus only does it appear that predication of the one kind or the other is possible with any two terms.

[2] The formal evolution of the Law of Non-contradiction into the Three Axioms has been a gradual process, brought to its consummation by the German logicians since the time of Kant. The laws of

16. The axioms, as universal laws of thought, must cover all possible cases. The case, also, which has been considered, deserved special notice; because the supposition of one

The bearing of the law on propositions considered as facts of naming.

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identity and difference have often been treated and expressed as one, oftenest called the Law of Contradiction; and, perhaps, the doctrine is most clearly apprehended when taken in this way.

The older logicians, as well as some of the more recent, seem to have frequently lost their way to the strict application of the law, through want of distinct thinking of the Quantitative Sign as an integral part of the term; an indistinctness which issued in the explaining away of numerical identity and difference into identities and differences specific and generic. The law of excluded middle, constantly and inevitably assumed and acted on, was kept in the background through the very facts of its palpability and its originally subjective obligation. Its formal introduction into logic as a separate axiom appears to be modern. Bachmann has collected a good many points in the recent history of the three axioms. (*System der Logik*, part i., sect. 2).

By Kant himself, and by several other German writers, there is added to the law of non-contradiction, as being also a logical law, Leibnitz's principle of the Sufficient Reason. There is unquestionable soundness in the objection taken to this addition by Sir William Hamilton, and by more than one of the Germans. If the doctrine means that nothing can *exist* without a sufficient reason, it is an assertion of the metaphysical law of causality; if it means that nothing can be *believed* or *known* without a sufficient reason, it is an assertion developed purely out of the laws of identity and difference.

The law of non-contradiction was neither generalized, nor formally planted at the root of the science, by any of the Greek logicians. (See Prantl, *Geschichte der Logik im Abendlande*, vol. i., 1855). Prantl, however, cites references by Plato to the law of identity (the bearing of which he questions): and he has made, from Aristotle, a large collection of passages which yield unequivocal assertions of all the three axioms; while, also, the law of identity is ex-

object, as the only thing given to be positively thought of, brings out, with a clearness not otherwise attainable, the primary idea of negation, as an explication of non-identity. That, in attempting affirmation with such data, we are driven on an assertion which is no real explication at all, is a fact not only to be accounted for easily, but leading us rapidly towards the development of the axioms in those cases for which the question of their use is important.

The datum of a proposition is, a relation between that which is denoted by the subject and that which is denoted by the predicate. But when we were required to think of A only, no predicate was given. For negation we found a

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plicitly declared by Aristotle to be the firmest principle of thought. The following, selected from Aristotle by Trendelenburg for his *Elementa Logices Aristoteleæ* (ed. 1852, §§ 9, 10,) are probably more marked than any other of Prantl's quotations:—" Τὸ *αὐτὸ ἅμα ὑπάρχειν τε καὶ μὴ ὑπάρχειν ἀδύνατον τῷ αὐτῷ καὶ κατὰ τὸ αὐτό. . . . . αὕτη δὴ πασῶν ἐστὶ βεβαιωτάτη τῶν ἀρχῶν· . . . . ἀδύνατον γὰρ ὁντινοῦν ταὐτὸν ὑπολαμβάνειν εἶναι καὶ μὴ εἶναι. . . . διὸ πάντες οἱ ἀποδεικνύντες εἰς ταύτην ἀνάγουσιν ἐσχάτην δόξαν.*" (*Metaphysica*, iv. 3). "*Δεῖ πᾶν τὸ ἀληθὲς αὐτὸ ἑαυτῷ ὁμολογούμενον εἶναι πάντη.* (*Analytica Priora*, i. 32; where the law is alleged as justifying the syllogistic reduction). "Ἀντίφασις δὲ *ἀντίθεσις ἧς οὐκ ἔστι μεταξὺ καθ' αὑτήν· μόριον δ' ἀντιφάσεως τὸ μὲν τὶ κατὰ τινος κατάφασις, τὸ δὲ τὶ ἀπὸ τινος ἀπόφασις.*" (*Analytica Posteriora*, i. 2). The express reference of logical rules to axiomatic principles has been traced to Galen. He, in his treatise *De Methodo Medendi*, states, as examples of such (*λογικαὶ ἀρχαί*), several axioms, for the explicit treatment of which he refers to his work *On Demonstration*, now lost. Among the instances he gives are these: the mathematical axioms, the law of causality, and the law of excluded middle; "*τὸ περὶ παντὸς ἀναγκαῖον ἢ καταφάσκειν ἢ ἀποφάσκειν.*" (See Prantl, p. 562).

predicate, by seizing on the supposition, implied in the oneness of the subject, that there must be other thinkable objects besides that, whatever it may be, which our subject signifies. But, the subject being, by the hypothesis, not only one, but something as to which we know positively nothing except its unity, a datum for real affirmation was wanting.

What would have sufficed to supply the want? Another Name, say B, for the thing denoted by the subject: out of this would have come the affirmation, "A is B."

Now, the interpretation to which a proposition thus generated is open, is one which may be put on all propositions whatever. Every affirmative proposition is equivalent to an assertion, that the subject and the predicate are but two different names for one and the same object, or group of objects: every negative proposition is equivalent to an assertion, that the subject is a name for one object, or group of objects, and that the predicate is a name for an object or group of objects different from the first. Besides being universally applicable, this is, of all interpretations of the proposition, that which is most purely formal; and, as being such, it has a peculiar aptitude for logical use. On this reading, the doctrine that affirmation and negation are assertions, respectively, of identity and non-identity, falls back into the class of truisms, if indeed it ever quitted or was in danger of quitting that class. A system of logical doctrines, seeking no further interpretation of the proposition, would be the nearest conceivable approach to a purely formal development of the science. We may often have occasion to recur to this reading, as the readiest means of showing how the special logical laws are only corollaries from the axioms.[1]

[1] "A proposition is a speech consisting of two names copulated,

Let us, in the meantime, test it by an example or two. "All logical doctrines are truths." It is meant, of course, not that logical doctrines are the only truths, but that they are some of those objects we call truths: "All logical doctrines—are—some truths." Plainly, the assertion is resolvable into this other; that the objects which we call "all logical doctrines," are the very same group of objects which we call also "some truths." Our names being assumed to be justified by the fact, every individual thing denotable by either name is denotable likewise by the other: the one

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by which he that speaketh signifies he conceives the latter name to be the name of the same thing, whereof the former is the name, or (which is all one), that the former name is comprehended by the latter." . . . . "An affirmative proposition is that whose predicate is a positive name, as 'man is a living creature;' a negative, that whose predicate is a negative name, as 'man is not a stone.'" (Hobbes, *Computation or Logic*, part i., chap. iii., §§ 2-6). On this interpretation of the proposition, as a Fact of Naming, Mr Mill, rejecting it as insufficient for founding the strongly objective position he is to take up, makes these remarks:—"The assertion which, according to Hobbes, is the only one made in any proposition, really is made in every proposition; and his analysis has consequently one of the requisites for being the true one. We may go a step further: it is the only analysis that is rigorously true of all propositions without exception. What he gives as the meaning of propositions, is part of the meaning of all propositions, and the whole meaning of some. . . . . If, then, this be all the meaning necessarily implied in the form of discourse called a proposition, why do I object to it as the scientific definition of what a proposition means? Because, though the mere collocation which makes the proposition a proposition, conveys no more meaning than Hobbes contends for, that same collocation combined with other circumstances, that *form* combined with other *matter*, does convey more, and much more." (Mill, *System of Logic*, book i., chap. v., § 2).

group of objects receives two different names, when it is considered from two several points of view. Again, "Logical doctrines are not paradoxes." Here we speak not only of "all" logical doctrines, but also of "all" paradoxes: we assert that the objects we call by the first name are not to be found anywhere among the things we call by the latter. The assertion admits this analysis:—The objects called "all logical doctrines," are non-identical with the objects called "all paradoxes." Each of the objects receiving the one name is an object different from each and all of the objects receiving the other: "logical doctrines" and "paradoxes" are names of two groups of objects, neither of which contains any individual object identical with any individual object contained in the other.

The bearing of the law on propositions considered as predicating attributes and classes.

17. The meaning of a proposition is not exhausted when it is read as a Fact of Naming. Perhaps every proposition has a deeper meaning. This is certainly true of all propositions which assert any knowledge worthy of analysis; and the relations which ground the higher kinds of inference cannot be fully theorized until the analysis is carried further. A name is not given without a reason; and almost every name intimates more or less fully the reason for which it was given.

All the reasons for names are resolvable into our considering of objects as Substances possessing Attributes; and, in respect of such attributes, objects are distributed into Classes, the names of which are Common Terms. The subject may now be itself the name of a group of objects constituting a class or a part of one; the predicate may be the name of an attribute which is possessed or not possessed by those objects. But, since the predicate is itself

also a class-name, there arises the further question, whether the class named in the subject is a part only of the predicate-class, or the whole of it, or no part of it at all.

In assuming even the applicability of two names to the same object, we had travelled far from the narrow nook of thought, which gave us, through the pure formulæ, our first glimpse of the developments receivable by the law of non-contradiction. We have travelled yet farther in assuming that each of the names is significant; and, when we regard the names as being, both of them, names of attributes, and through these of classes, we have reached the most cumbrous of the complications under which the question of identity or non-identity can be contemplated.

If we stop short at the point which exhibits the subject-term as being the name of a substance, or of a group of objects considered as substances, while the predicate is regarded merely as being the name of an attribute possessed or not possessed by that object or objects, we may appear for a moment to have lost our way. The question, whether an object possesses or wants an attribute, is not very obviously resolvable into the question, whether the subject is or is not identical with the predicate. But even the most common expressions yield this interpretation; and the frequent shortcoming of the predicate—its expression through an adjective—is rapidly supplied, both in thought and in expression, when we take the further step of regarding the predicate as being the name of a class. Both terms of the proposition may now take the form of substantives; both, if common terms, may be taken as names directly denoting groups of objects, and only implying the attributes in respect of which the class-names are given.

If we assert that "All men are imperfect," the full meaning of the allegation is, that all men are a part of the class "imperfect beings;" that, in other words, "All men—are —some imperfect beings." If we assert that "No men are unimprovable," we signify that no men are any part of the class of unimprovable beings; that "No men—are—any beings unimprovable." In the one case we assert that the objects called, when thought of with reference to a certain attribute, "all men," are the same objects which, when thought of with reference to a certain other attribute, are called "some beings imperfect;" in the other case we assert, that the objects receiving the name "all men," are non-identical with "all" the objects receiving the name "beings unimprovable."

In a word, the relation of identity or non-identity, with the determination of thought towards the assertion of either the one or the other, covers all the complications, various as they are, which are made possible through the mutual ramifications of classes, as included one in another, wholly or partly, or as mutually excluded in whole. We have only to presuppose the correlation of Whole and Part (a correlation not arising in the comparison of individuals); to watch carefully, as to each of our common terms, whether as used in our propositions it denotes all, or only some, of the objects denotable by it, or constituting the class it signifies; and to remember that, of each term in its relation to the other, the words indicating whole or part ("all," "any," or "some"), must be thought as integral parts. These precautions being taken, predication through common terms is interpretable as an assertion of the relation of identity or non-identity, with the same ease as that which

we find in so interpreting predication through singular terms.[1]

The necessity of the axioms for the unity of logical science.

18. Exception has frequently been taken to the formal statement of the law of non-contradiction as the one central doctrine of logic. It is not alleged that the law is either deniable, or so much as doubtful; but it is said (and this is the objection most frequently urged), that it is a mere truism, a truth so obvious as not to deserve explicit notice.

The same charge may be brought, with equal fairness, against the geometrical axioms; and these might be treated as the logical axioms have so often been. The truism, the "trifling proposition," that "Things which are equal to the same thing are equal to one another," might be refused a formal place in geometry; and the student might be invited to supply for himself it and its fellows, in the demonstration of those initial theorems for which no derivative ground had as yet been laid down. Perhaps the practical evil would not be heavy; but the symmetry and coherence of an exact science would be annihilated.

When the logical axioms are refused their legitimate place, the mischief worked is incalculably greater than any

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[1] It is especially to be observed that, when the quantitative signs are accepted as integral parts of the given terms, the identity or difference of the objects is strict and literal. We are thus rescued from all necessity of loosely translating identity and difference into likeness or unlikeness, or of instituting fine distinctions between identity specific and identity individual, between identity total and identity partial; from all those artificial expedients, in short, which have often perplexed so seriously the theory of inference, and made it so difficult to trace the laws of the process upwards to the one central principle.

that could arise from a similar procedure in mathematics. There have been constructed very many logical systems, which are quite adequate for the testing of any argument that could be given, and which yet want, not only the formal statement of the axioms, but all express reference to them. A science so treated cannot fail to lose much of that systematic coherence, which is the scientific and philosophical characteristic; and no science loses, through such treatment, more of that character than ours. It becomes an aggregate of theorems which are really derivative, but which, not being centralized in their common source, not only exhibit no apparent unity or correlation, but degenerate (a weakness incident to a science so exclusively formal) into mere technical rules, usable and used without conscious reference to any principle at all.

In a word, the construction of logic through secondary laws exclusively, does and must destroy, or seriously impair, its unity as a science. Consequently, there is thus injured, likewise, the evidence of its speculative validity as an analysis of predication and inference. It is yet a worse evil, that this course of treatment diminishes largely the value of the study as a philosophical discipline of thought. In justice alike to the science, and to ourselves its students, unity of system, and consistent development of doctrines, should be steadily aimed at; and it must firmly be maintained that this purpose can neither be reached, nor so much as approximated, unless the law of non-contradiction be expressly laid down as the axiomatic foundation, and unless, also, there be expressly resolved into that law all doctrines whose dependence on it is not self-evident.

19. It is possible to test the validity of every inference,

Development of the axioms in reference to terms interpretable.

by a direct analysis of its propositions as assertions of identity or difference. For some of the purposes which the science is designed to serve, such analysis would be quite sufficient. It might, for instance, yield an exhaustive and competent theory of immediate inference. But it would not enable us thoroughly to theorize the syllogism. Syllogistic arguments given might be adequately treated in this fashion ; but the character of the syllogism as a representation of several distinguishable processes of thought could not be efficiently displayed, without exhibition of those objective modifications under which the relations of the syllogistic elements come to have place.

With a view to such developments, and also for another reason, the three axioms will here be presented in one or two of the shapes which they may assume, when they are considered with pre-supposition of interpretation of the terms used in propositions.

The other reason is this. The doctrine, that inference is merely an explication of the implied, although it is a truth both undeniable and instructive, is a truth which is far from being palatable. When we are first asked to make ourselves familiar with it, we are apt to forget how mighty is the difference between implication and explication. It costs us an effort to become convinced that the difference is that between obscure thinking and thinking that is distinct—between a thought which is isolated in consciousness and a thought of which we are conscious as an element in a system ; that it is the difference between impotence and power, between a cloudy dawn and a sunny noon-day. It is desirable, then, to place this difference in full light.

Now, the contrariety of character between a thought implied in the subject of a proposition and the same thought

evolved in the predicate, has been anxiously brought out by the framers of those forms of the axioms which are here selected from many others. The concrete character of the cases contemplated in these theorems causes difficulties of expression, which make it desirable to give alternative views, both for avoidance of mistake, and for the suggestion of reflection.

(1.) Whatever is implied in the signification of a term given as the subject of a proposition, may, as predicate, be explicitly affirmed of the subject. Any notion which is implicitly thought in the subject may be explicitly affirmed in the predicate. Of any object or objects denoted by the subject, there may be affirmed in the predicate any attribute consistent with our thought or notion of the subject.

(2.) Whatever is inconsistent with the signification of a term given as the subject of a proposition, may, as predicate, be denied of the subject. Any notion, the contradictory of which is implicitly thought in the subject, may be explicitly denied in the predicate. Of any object or objects denoted by the subject, there may be denied in the predicate any attribute inconsistent with our thought or notion of the subject.

(3.) Of any term given as the subject of a proposition, any other term must be either affirmable or deniable as predicate. Of any object or objects denoted by the subject, any attribute whatever must be either affirmable or deniable in the predicate.

The objective relations of predication and inference through common terms.

20. It is only for the sake of predication and inference through common terms—for the sake of processes explicating the relation between class and class of objects compared in respect of diverse attributes—that logic is worth elaborating into a scientific shape. Knowledge worthy of the name

—knowledge the acquisition of which is a duty adequate to the capacities of intelligent beings—knowledge fitting man to act, imposing on him responsibilities, and enabling him to merit rewards—is a knowledge of the attributes of objects, of the laws by which they are governed, of the compass of those several laws, and of the fine and manifold relations in which, through likeness and unlikeness of law, man is placed towards man, and each man towards nature and the Power that governs it and him. Our knowledge of individuals is clear and bright, and shines out spontaneously through intuitions, which dawn on us without our seeking; but the light which thus we see, illuminates a region within which rational life has hardly begun to germinate. Our knowledge of laws is reached only through self-determined energy, through struggles to emerge from doubt, and contendings against error, and slow and painful ascent from height to height of cognition. But, while we do struggle, and contend, and rise, the horizon broadens round us, and our mental vision gains new strength and delicacy from exertion. The idea of law itself passes into that of causality: objects which obey law do so either as causes or as effects. Out of causality again emerges the great idea of purpose; for purpose is preconceived effect, and the efficient cause becomes operative towards this effect as means. Purpose carries us upward, through cause, into the sphere of mind, of thought and will as attributes of beings capable of designing; while here we find ourselves to have adventured into a field of inference, widening our view as we advance, till we have reached the contemplation of one overruling Purpose, of which perceived objects, and discovered laws, and physical causes, and human will, are but the exponents, and consequents, and ministers.

In all the paths which mind can traverse, logical laws are operative as prohibitions guarding against divergence. Logic is concerned, not with the matter of thinking, but only with its forms. Over these, however, it holds exclusive sway. And there is a necessity for the exercise of its powers,—a necessity which becomes the more pressing as the known relations of objects grow wider and more complex. The law of non-contradiction could not be violated at all, were it not for the need we lie under of thinking through words, whenever we do think of any thing that is not individual. If every object had but one name, violation of the law would be practically impossible. It is because every object has many names, that the natural course of thinking betrays us into judgments in which the law is unconsciously broken. It is because of the intertwining and often conflicting relations of all thinkable objects, that words are so apt to be used as symbols for thoughts which they do not clearly represent; and the more various and extensive the relations are, the more imminent becomes the danger of self-deception. Therefore it is that logical laws are valuable, not to supply matter for thought, but to test the genuineness of thought, and to protect thinking from being disguised through its expression.

21. Logical laws are the scaffolding which gives support to derivative knowledge in the course of its construction. When the structure has been completed, they become for us the plummet and level, through the use of which may be determined the firmness or instability with which it bears on its foundation. But logic does not, and cannot, carry a single stone to the building.

The relations of logic to truth.

It enables us to explicate, not the relations in which ob-

jects exist, but only the relations in which they are thought. The attempt to fix the truth or falsehood of any one proposition given in isolation, is not more palpably extra-logical, than is that of incorporating into the science principles really metaphysical or ontological, that is, bearing on the universal relations between knowledge and existence. This consideration justifies and commands the positive exclusion of all such doctrines as those of the Categories, and of Modality in Propositions. A prohibition which there may be a greater risk of disobeying, is that which excludes all questions as to the objective truth of given classifications, that is, as to the relations actually connecting or separating the objects designated by given common terms. All judgments given for logical analysis, are, for logic, virtually hypothetical. The objects thought as constituting the class may be non-existent; the law through which they are combined may be imaginary; some or all of the objects may be exempt from the law. It is always important, it is often unspeakably so, that we should learn whether it is true that none of these negations has place, and whether therefore a given class-name implies a fact of real and positive knowledge. But this is a question to which, in all its parts, logic stands resolutely silent.

The science must, indeed, look abroad on those objective conditions, those relations between thinking and that which is thinkable, by which the human intellect is fenced in, round and round. But it asks only how those conditions modify the manner of thinking; it takes account of none of these but such as do necessarily determine thinking towards one or another of its only possible forms: and it scrutinizes them for no further purpose than that of eliciting and explaining those forms. When the astronomer looks down from his

watch-tower, he is pleased and grateful to see how the sun illuminates the earth, and diffuses life and gladness over the expanse of animal and vegetable nature; but his duty is that of surveying the heavens, and discovering the laws which guide the stars in their courses. With no less satisfaction does the logician perceive that truths, good for man, are revealed in those intuitions on which all thinking rests; but it is no part of his function even to assert those truths, far less to justify or systematize them.

Thus is Logic placed towards all those principles which it either developes, or assumes as given. The correlation of identity and non-identity is itself a law metaphysical as well as logical, a law of existence as well as a law of thought; but it is only in the latter aspect that it is logically important. So is it as to all those other relations, without the assumption of which the law of non-contradiction cannot be developed. Number, quantity, whole and part, are sufficiently treated for our purpose, when they are regarded as conditions determining the forms in which objects are thought of. Perhaps, again, the relation of substantiality covers, in logic, a wider ground than any other of those modifying conditions. But the most paradoxical or sceptical denials or doubts as to this relation would leave, untouched, the formal view which we have to take of substance and attribute, as being actually correlatives, and thinkable only together, but as admitting of being thought from either of two opposite points of view, which give prominence alternately to the one and to the other.

In short, Logic seeks to develope one principle only—the central Law of Non-contradiction. It developes that law with reference to certain modifying principles: but these it assumes only as given in actual experience, and does not

seek to develope; and, further, it assumes them only as being (what they undeniably are) psychological laws, laws regulating thought—declining to inquire into their ontological character as laws of being.

'he postu-ates of lo-ical cience.

22. The desire of clearly illustrating drives us often, in logical writing or teaching, on exemplification through propositions whose terms have meanings known to those we address. There is a danger in this. The truth or falsehood of each of the propositions being thus known, the mind is allured away from the logical question, whether one proposition does or does not follow from another. Symbolic terms, of the algebraic type, are, in spite of their dryness and repulsiveness, by far the aptest for logical examples.

We cannot see distinctly what the problems are which the science is able to solve, until we consider it as working on materials of this indeterminate character.

Logic neither undertakes nor requires any interpretation of given terms. It is bound to deal with terms which may mean any object whatever, and which are not given as the names of any fixed objects. But no terms can be treated by logical re-agents, unless they are given in a shape that fits them to the crucible. Every term must signify something; and logic cannot deal with terms unless there be given to it the minimum of their signification. The science, like every other, has its *Postulates*.

The Postulates are specificated sufficiently, when they are stated with reference to the narrowest data. They are two. Logic must require, as preliminary conditions of its activity, answers to one or both of two queries bearing on every given term. *First*, Is the term singular or common? *Secondly*, If the term is singular, no further information is

required. But, if it is common, this other question must be put: Is it in the given case distributed or undistributed?—Is it used in the whole of its extension, or only in a part of it? Both queries bear on the Quantity of the terms, the question which, as we have seen, arises secondarily in logic. If we had to deal only with the Quality of propositions, which is the primary logical question, it would not be necessary to put them.[1]

Both queries are, to a certain extent, answered by the forms in which assertions are usually couched, when these have, as in ordinary speech, terms of fixed signification. But, when they are not so answered, the answer must be sought, that it may be incorporated in the expression of the terms. Singular terms require and receive no Quantitative Signs. Common terms do require either "all" ("any") or

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[1] Both of these demands are virtually embraced in Hamilton's *Fundamental Postulate of Logic:* "That we be allowed to state in language what is contained in thought." (Baynes, *New Analytic of Logical Forms* (1850), p. 4). There will hereafter be much of reference to the opinions of that distinguished thinker and profound scholar. Therefore it may be well to say, here, that the present writer's acquaintance with them is derived exclusively from the outline just referred to, which was published with Sir William Hamilton's sanction; from Sir William Hamilton's volume of *Discussions* (1852); from incidental notes in his edition of Reid (1846), especially those on Reid's *Account of Aristotle's Logic;* and from a few observations furnished by him to the last edition (1854) of Mr Thomson's *Outline of the Necessary Laws of Thought.* It is to be hoped that the promised publication of Sir William Hamilton's Lectures will speedily furnish information, of which, in regard to points not a few, the students of his masterly logical system are still very much in want.

"some," and should have the one or the other, whether they be subjects or predicates.

The postulates are reasonable. They do not stretch a step beyond those two objective conditions (individuality and universality), by the one or the other of which actual thinking is formally modified. The logician is bound to provide laws applicable to terms whose meaning is as arbitrary as that of algebraic symbols. But the algebraist, too, has his formal postulates. His *a*, *b*, *c*, and *x*, *y*, *z*, are thus far fixed in signification, that all of them denote numbers; and he is warned, by pre-arranged marks, whether the numbers are integers, or fractions, or powers. Our terms, even though symbolic, are thus far fixed in signification, that they must denote possible objects of thought, and objects thought under one or another of certain conditions. In demanding what the given conditions are, we ask for explanations exactly parallel to those which are allowed to the mathematician. No narrower pre-information will suffice, if logic is to be anything better than a theory of dreams.

'he formal imits of ›gical analysis.

23. The postulates being granted, let us ask, lastly, what data, furnished with them, give a hold to laws purely logical. The relations of common terms are the only ones with reference to which it is worth while to generalize the cases.

First, The narrowest datum on which logic can work without foreign aid, is one proposition,—the assertion, through a copula, of identity or non-identity between two terms quantitatively determined. From such a datum, the science can regulate and justify the evolution of certain other propositions by Immediate Inference.

Secondly, Logic can work with incalculably greater freedom on two propositions given. These are data for the

normal form of Mediate or Syllogistic Inference. (1.) The data may not yield, by inference, any third proposition. If so, the reasons of the failure can be shown. (2.) The data may yield, by inference, a third proposition. If so, the science can direct the evolution of that proposition, and assign reasons both why it is evolvable, and why no others are so. (3.) While, in both cases, the reasons are ultimately traceable to the logical axioms, it can be shown, through derivative laws, that the result depends immediately on the question, whether the given terms do or do not constitute a series, related to each other both in Extension and in Comprehension.*

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* Something, perhaps, should here be said, of the reasons which have seemed to justify the raising, in the present chapter, of questions in regard to the function and limits of logical science. Few or none of these needed to have been touched on, if it had been sufficient to regard logic exclusively from the practical side. All of them (and, it may be, others also) imperatively demand attention from those who would form a right estimate of logic as a system of speculation, —those who would know what value it has in itself as an exposition of the regulative theory of human thought.

In this country, there has never been seriously contemplated the possibility of a Logic absolutely pure or *à priori*, that is, of a system of logical science not only thoroughly demonstrative in its deductions, but not acknowledging even any data that are empirical. The possibility was broadly averred by Kant; and the endeavour was made to work it out in not a few German works, among which may be named especially those of Kiesewetter and Hoffbauer, and the symmetrically systematic treatise of Twesten. Gradually it came to be perceived, that even the ablest thinkers who had taken up this position, had not been able to proceed a step without silently assuming empirical or psychological data. Those earlier writers who exhibited the fullest proofs of this assertion were Troxler and Bachmann. The attack on the Kantist limitation of the sphere of logic was next

undertaken, on much deeper philosophical principles, by the same energetic iconoclast, of whom, not very long ago, Rosenkrantz complained, that he "had brought philosophy (that is, Hegelism) to a stand-still." In Trendelenburg's *Logische Untersuchungen* (1840), every inch of ground was cut away from under the feet of those logicians who aimed at constructing the science without presuppositions. Yet the writings of this singularly acute and learned controversialist are not the only symptoms indicating that, in Germany itself, the reaction has issued in an oscillation stretching equally far from the truth on the other side. It is not easy to see how Trendelenburg himself could frame, in consistency with his leading opinions, a positive theory of predication and inference which should be anything else than a hybrid generated between logic and metaphysics. The instructive treatise of Drobisch (*Neue Darstellung der Logik,* 2d edit. 1851), also incorporates objective elements so freely, and brings them to bear on the formal laws of thought with such intimacy of relation, that the latter are fairly overbalanced, and the science ceases to be operative as yielding readily practical tests for explicative thought.

But Drobisch's mode of working out the details does seem not to be necessitated by his own opinion as to the function of the science. A paragraph of his preface, explaining that opinion, may serve to illustrate the position which, here and afterwards, it is endeavoured to make good in the text.

"Trendelenburg says, of the formal logic, 'that it desires to understand concept, judgment, inference, from the self-referred activity of thought; that hence it separates thinking from its object, as if the mirror which receives the light were separated from the ray which falls on it; but that such separation is hazardous, since the law of reflection is not conditioned by the mirror alone.' This view is incorrect.

"Formal logic does not presuppose a pure thinking, and does not undertake to analyse or develope the forms of such thinking in the abstract. Its presupposition is that concrete thinking which is in the most intimate union with cognition. From such thinking, the science, through abstraction, gains its fundamental forms; and

then, according to the laws yielded by consideration of the relations of the forms, it connects the forms with each other, and thus reaches derivative forms. Forms without matter or content, logic does not know; it knows only those forms which are independent of the special matter that may be placed in them, and for which, therefore, the matter, although it can never be dispensed with, remains indeterminate and accidental.

"The fundamental forms of thinking are gained in a manner like that which yields the fundamental forms of geometry. These are only the remainders, which abstraction leaves over from the physical and chemical properties of bodies perceived through the senses. The idea of empty space is an abstraction, foreign both to sensuous intuition and to its reproduction in memory; the geometrical surface requires a second abstraction; the line and the point require a third and a fourth. In like manner does logic arrive at the concept, its marks and its relations. But geometry is not contented either with the discovery of the fundamental forms, or with the classification of corporeal forms as presented by experience: through combination of the fundamental forms, it reaches ideal constructions, in which indeed it partly reconstructs that which is given and actual, but partly comes on formations which appear to us like strangers in the known world of sense. In a manner exactly similar—in the doctrines of judgment and inference, of divisions and proofs—does logic deal with the fundamental forms of concepts; while it allows itself to be guided by nothing but the consistency of the forms of thought with each other, the consistency of thinking with its own principles. This consistency is the only logical truth."

Of recent English works, two should be particularly referred to, as placing the function of logic on a solid and philosophical basis: to both of these more obligations than one are here due; Mr Karslake's masterly sketch, the *Aids to the Study of Logic*, book i.; "Pure Analytical Logic" (1851); and Mr Mansel's treatise, alike acute and comprehensive, the *Prolegomena Logica* (1851). Much of valuable suggestion in regard to principles is furnished also by Mr Chretien's *Essay on Logical Method* (1848). Mr Moberly's *Lectures on Logic* (1848) may be advantageously consulted for

several points of special doctrine; and Mr Kidd's *Delineation of the Primary Principles of Reasoning* (1856), is exceedingly instructive, both in its original matter, and in its analytic comparison of recent logical systems.

This may be as fit a place as any, for alluding to the mathematical theories of thought, of which several have been propounded by actively thinking men, both in other days and in our own. Some of these have been content with expressing logical doctrines and rules in mathematical forms: others have insisted on seeking the foundations of logical science in principles really mathematical. To the valuable German treatise just quoted from, there is annexed a "Logico-Mathematical Appendix," in which long and complex trains of reasoning are designated by algebraic symbols. The reduction of all thinking under an elaborate series of symbolic formulæ is the design of an exceedingly subtle and able work, Dr Boole's *Investigation of the Laws of Thought* (1854). In Mr De Morgan's *Formal Logic* (1847) the practical side is often approached very closely, and the pure laws of thought are developed in several of their relations with very great skill; but the principles of logic are thoroughly subordinated to those of mathematics.

All attempts to incorporate into the universal theory of thought a special and systematic development of relations of number and quantity, must be protested against with equal firmness, whichever side of the question we may look to. If the systems are to be estimated speculatively, as philosophical expositions of the laws which regulate all thinking, it must be said that they are faulty both by defect and by excess. They endeavour to theorize all thinking, by examining thought only as exerted on one kind of objects: they allege, as bearing on thought universally, laws which rule it only in certain cases. If, again, the systems are supposed to furnish rules available for practice, they must be pronounced to be both unnecessary and ineffective; and this objection lies, not only against the intrusion of mathematical principles, but also against the adoption of mathematical forms for any purpose beyond that of incidental illustration. No cumbrous scheme of exponential notation is needed, and none such is sufficient, for the actual guidance of

thought when its objects are not mathematical: when its objects are so, the science of mathematics is both bound, and is the only science that is qualified, to yield the principles on which rules may be founded. The question may be considered, also, with reference to the value of logical study as a discipline of the mind. Now the mathematico-logical theories tend, one and all of them, and tend the more strongly the nearer they bring their rules towards the forms of the higher mathematics, to convert logical study into a mere cramming of the memory with formulæ, and logical practice into a mechanical manipulation of rules not known to have reasons. Even in its genuine shape, the science is, on account of its formal character, liable to both of these dangers; and the duty of its expounders is to guard against them, not, certainly, to run wilfully into their way.

PART FIRST.

THE DOCTRINE OF TERMS.

The signification of terms, singular and common.

24. We have already learned that, and how, the two aspects in which objects may be regarded, as individuals or as members of a class, impose on judgments and propositions the only objective modifications to which they are universally subject. The Terms through which objects are thought must be either Singulars or Universals. It remains only, in reference to terms considered as elements of judgment and predication, to place their characteristics in the position in which they are directly available as the foundation of logical rules. Here the principal question is that of the manner in which both images and concepts, and also the relations of these to each other, are denoted by words.

Singular terms call for little examination. Common terms must be scrutinized very exactly; both by reason of the difficulties they involve, and because inferences through them, being the highest formal developments of thought, are the processes whose laws constitute the highest sections of logical doctrine.

For almost all logical purposes, it is sufficient to consider terms as the Names of Objects of Thought. Singular terms are thus names of objects thought of as individuals; common terms are the names of classes thought of as constituted

by individuals or kinds of individuals. But we must always hold ourselves prepared to fall back, when it shall be necessary, on these two limiting facts:—first, that logic, both in assuming data and in working out results, has regard, not to the question whether objects are real, but only to the question whether and how they are thinkable and thought of; secondly, that terms denote objects only as thought of under some given relation.

The words which constitute terms.

25. The relations under which we can think of objects may be either comparatively simple, or indefinitely complex. Further, the expressibility of a relation in few words, or its requiring of many, may frequently be determined by circumstances extraneous to the character of the relation.[1] Therefore, a term may consist either of one word, of several, or of many. But its one word, if it has no more, or its leading word—that which expresses the most prominent idea of a group—must, admittedly, in the first place, be a noun, either substantive or adjective; and, next, it may rightly be held, that a common term does not bring out completely the concept of which it is a sign, unless its one word or leading word is specifically a noun-substantive. An adjective, indeed, does often do duty as a predicate; but a substantive is required for giving easy and full expression to an idea constituting a subject. Now, a given proposition is not adequately developed, unless it has a form enabling us,

[1] Very many short or simple terms, both ordinary and technical, imply ideas which are exceedingly complex. Such terms are conventional abridgments, adopted for the acceleration of thought, as well as of speech; and they are of constant occurrence as names of objects possessing universal interest and importance. They carry with them both advantages and dangers.

without interpolation, to extricate from it all its possible results; and some of these results are not attainable unless through transposition of the terms.[1]

[1] A word or phrase which may by itself be a term, is said to be Categorematic (κατηγορεῖν, to predicate); one which cannot, is Syncategorematic. By a noun is meant a noun in its nominative case: the oblique cases are excluded, with all other parts of speech. Many logicians take this distinction; that a substantive is required as subject of a proposition, but that an adjective may be logically accepted as the predicate. According, however, to the view stated in the text, such an expression as "Some men are good," is elliptical, and should, for logical treatment, be explicated into "Some men are good persons." In such fillings-up, we are doubtless exposed to the risk of limiting the predicate to a meaning narrower than the datum; but this is not difficult of avoidance. There are strong reasons for insisting on the doctrine, that substantives are the only parts of speech truly categorematic. "The predicate as predicate carries with it the mark of dependence; it does not become a free concept, till it assumes the form of substance, and may in this form become subject." (Trendelenburg, *Logische Untersuchungen*, ii. 144; see also Ritter, *Abriss der Philosophischen Logik* (1829), p. 68.) When we set about bringing logically to light the relations of given terms, we are not entitled to suppose that each of our terms will continue to discharge the function it had in the proposition which gave it to us; and we are bound, in setting forth our data for logical manipulation, to give to each of their elements a form which shall bring out its character as fully as possible, and qualify it for discharging any function which any possible variety of inference can impose on it. The imperfection of the evolution which the adjective yields is exposed as soon as we attempt logical conversion. The example, as above expanded, furnishes at once the converse,—"Some good persons are men;" but, as first set down, it would not yield any intelligible converse, unless either the substantive were interpolated in the process, or the proposition thrown into one or another of those abstract shapes, which, as we shall see, are almost utterly unmanageable.

The manner of signification of words constituting terms.

26. We must now note, in a general way, the manner in which the singular and the common term severally signify the individual and the class.

(1.) Nouns may denote individuality in any of several ways. They may be proper names; as of persons or places: "Napoleon, Socrates; England, Edinburgh." Only it should be noted that, names of persons having long fallen short of the demand, many or most of such words are really common terms, though interpreted as singulars, through our knowledge of the circumstances in which they are used. The words, again, may be words which are strictly common terms, but whose meaning is individualized by the accompaniment of definitive descriptions: "The man whom I saw yesterday;" "the meadow which lies before my window;" "the argument by which you convinced me." Nor will a term be the less a singular, though the descriptive addition be by itself insufficient to indicate, or may even leave it uncertain to the speaker himself, what individual it is meant to designate. "Yonder hill," may require a gesture to determine the one hill intended; "the most profound philosopher of our age," may be a name for a person undetermined by those who use the phrase.[1]

Another class of cases is more apt to be mistaken. A thing is thought of as an individual, whenever it is thought as one object, although its unity should be made up of several or many individual parts; and any given individual may itself be next thought of as a part of some other

[1] An object indicated thus indefinitely was called by the old logicians an *individuum vagum*. It was a disputed point whether the term denoting it were properly a singular, or a common term used particularly.

thing, which in its turn is thought of as an individual. Thus, among objects of perception, we may think successively of "that trunk, that tree, that forest:" and so, likewise, may it be for phenomena of reflective consciousness:"—"The idea I have at this moment, the judgment of which that idea was an element, the course of reasoning in which that judgment was one of many steps." Examples like these, which would fall under the scholastic description of an integral whole, are not the only ones. Even the logical whole, that is, the class constituted by the individuals designated by the common term, may itself be abstractively thought of as one object. The distributive all (= each), leaves the common term as the sign of a true concept; the collective all (= all taken together), transforms it into the sign of an individual unity.[1]

(2.) In regard to the manner of signification in common terms there is required one remark only. Common terms are the names of classes, constituted, either immediately or through intermediate steps, by individuals, which are thought of together in respect of their possession of certain common attributes. Sometimes the term distinctly states the attribute; but, even so, the objects or substances continue to be, in speech as in thought, more prominent than the attributes. Much more frequently, and especially when the objects constituting the class are many, the common term is an arbitrary name, which briefly, but directly, denotes the class, and is applicable to each and all of the objects, while it merely implies or connotes the attributes. Indeed, both in common life and in science, it is much oftener easy, from

[1] See, among other explanations, that of Wallis, *Institutio Logicæ*, part i., cap. ii.

a large combination of obvious characteristics, to place objects in a certain class and give them a conventional class-name, than to fix with strict accuracy the attributes which are essential and peculiar to the class.

The quantity of terms.

27. Out of the essential distinction between the two kinds of terms, there arises a broad distinction in the manner in which they are severally usable.

Individual objects are thinkable only as indivisible units. Consequently, a singular term can never, without altogether losing its character, denote anything less than the object of which it is at first assumed as a name.

Classes of objects, on the other hand, may be thought either in whole or in part. We may think, either of all the objects constituting the class, or only of fewer than all of those objects; and we cannot think the concept, or use the class-name, otherwise than for thinking and expressing either the one or the other of the two alternatives. A common term *may* be understood as denoting all the objects of the class; it *does*, in every proposition, denote either all or fewer than all of them.

A common term, when it is used to denote all the objects of the class, is said to be taken Universally, or to be *Distributed*; that is, to be spread over the whole class, or to be applied to all the objects distributively, not collectively—to each, not to all together. A common term, when it is used to denote fewer than all the objects of the class, is said to be taken Particularly, or to be *Undistributed.*

A common term, therefore, may, in a useful sense, be said to have *Quantity*; its quantity being variable, as universal or particular. It is scarcely correct, and not at all useful, to consider a singular term as having quantity. When it is

said that a singular term is equivalent to a common term distributed, all that is meant is, that it does not admit of non-distribution or particularization.

We must be able, then, whenever a common term is given, either to assume or to infer whether it is distributed or undistributed. The state of the fact is, for the subject at least of a proposition, indicated in common language, by a variety of prefixed phrases. Two of these are, for convenience, used always in logic; and, it may here be observed, the signs should, for logical working, be prefixed to the predicate as well as to the subject.

The signs of the distribution of common terms.

28. The ordinary Signs of Distribution[1] are fully interpretable for us, because the universality of terms has no degrees, and is Definite. The number of individuals contained in a class may indeed be, and almost always is, indeterminate; but, be they few or many, we do, in distributing the class-name, definitely embrace all of them under it.

All logicians adopt, as the sign of distribution in affirmative propositions, the one prefix "all" or "every." Some adopt it for negatives also; but such a use of it is apt to mislead. In the expression "All X's are Y's," the subject is understood by every one to be distributed; but in "All X's are not Y's," most persons would interpret the sign as intimating non-distribution.[2] It is safest not to in-

[1] They are such as these: "all, every, any, whatever." Both of the articles, too, are so used when joined with ampliative phrases; as, "The man (or a man) who has true self-respect, is not likely to refuse due respect to others."

[2] "All X's are not Y's," is naturally understood as being merely a denial of the assertion that all the X's are Y's; as equivalent to

cur the risk. It may be avoided by using the prefix "any" in negation. This sign is useable without any difficulty for the predicate; and it is best to use it as a prefix for the subject also. If there is thought to be a needless awkwardness in such phrases as "Any X's are not any Y's,"[1] we might content ourselves with an indesignate subject, and say, "The X's are not any Y's;" a form which by most hearers would be interpreted as distributing the subject, while it may, at all events, be taken in that sense by agreement.[2]

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this: "It is not true that all the X's are Y's." But we are entitled to make this denial, if it be true that even "Some of the X's are not Y's," or that "There is some (or any) X which is not a Y;" and this is all the meaning we commonly attach to our "All X's are not Y's." In so interpreting it we are, as in numberless other instances, working out logical doctrines without being aware that we are doing so. "Some X's are not Y's" is logically the contradictory of "All X's are Y's," that is, a proposition necessarily inconsistent with it.

[1] Ordinary language would give this arrangement:—"Not any X's are Y's;" and this again would pass into "No X's are Y's." These forms, and especially that which both displaces and incorporates the negative sign of the copula, are apt to tempt us into mistaking, for a moment, the quality of the proposition, and supposing it affirmative. The alternative afterwards proposed has the opposite fault: it conceals the quantity, and might lead us to suppose the proposition particular.

[2] Propositions whose subject has no prefix of quantity are usually called Indefinite, more properly (Hamilton) Indesignate. The subject, if a common term, must necessarily be either distributed or undistributed; and logicians are wont to say that we cannot decide which of the two it is, until we have interpreted the terms, and considered the matter of the judgment. But, when such a proposition is negative, probably no man would dream of interpreting it particularly; and even when it is affirmative, we do certainly tend to give the same interpretation. On common talk,

The signs of the non-distribution of common terms.

29. The ordinary Signs of Non-distribution[1] are not fully interpretable for us. Each of them, besides directly signifying particularity, does also denote or imply some closer specification; and the particularity is the only part of their meaning that yields an idea logically available.

The only quantitative distinction with which a universal theory of thought can deal, is the all-pervading distinction between "all" on the one side, and "not-all," or "fewer-than-all," on the other; between a whole on the one side, and, on the other, something of which we can say only that it is some part or other of that whole. Accordingly, all the ordinary signs of particularity must, so far as logically cognizable, be taken as equivalents; and, for convenience, all of them are translated into the technical "some." Were it not that even one of the parts of the logical whole is enough to let in the logical "some," it would be exactly parallel to

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or on oratorical or poetical effusion, it would be unreasonable to impose very severe restrictions. But in argument it would hardly be unfair to insist on interpreting universally, as against an opponent, every indesignate proposition he adduces. A reasoner who expresses particular assertions without explicit limitation, must do so either because he designs to be ambiguous, or because he thinks confusedly, or because he is (perhaps unconsciously) suppressing some step of reasoning which it would be right to force out into xplicit statement. Sanderson places such assertions among his "Suppositiones," or propositions implying others; and he interprets his example as a disjunctive: "A ship is necessary for crossing the strait; that is, this, that, or the other ship is necessary." (*Logicæ Artis Compendium*, lib. ii., cap. 2.)

[1] Some, a few, a very few, few (= many not), a great many, not a few (= many), most, a small, large, or considerable number, a majority, a minority, a small or large proportion, nearly all, all but a few, more or less than half, &c.

one popular phrase. Our "Some X's are Y's," were it not that it might possibly mean only "One X is Y," would be equivalent to the assertion, "There are X's which are Y's."

The true character of Logical Particularity requires to be very precisely understood. It is in all respects Indefinite.

(1.) In the common use of words, a proposition introduced by any of the limitative signs is (unless accompanied by an explanation) understood naturally and fairly as a proposition implying another. Our usual "some" means "some, but not all;" or, "some at most." If we explicitly assert that "Some are —," we are understood as implying that "Some (or many) are not —." No man asserts merely of "some" if he might assert of "all."[1]

The "some" of logic is equivalent to "some at least;" "some, it may or may not be all." And why? Because this is the minimum of signification bearable by any limitative sign of quantity. So much as this must be signified by each of them; so much may be assumed as involved in every assertion of the sort; and if, in a given instance, more is signified, the overplus may and should be treated as a separate proposition.[2]

(2.) All the ordinary signs are more or less definite in

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[1] If I say, in common phrase, that "Some men are wise," I am understood to imply an opinion, that some men, or many, are not wise.

[2] If I use the "some" in its logically restricted sense, and thus design to convey no implied meaning at all, I may say that "Some men are mortal;" since I know that "All men are mortal," an assertion by which mine is covered. If I were to speak with implication of the usual *annexum*, my reason for saying that "Some men are mortal" would be, that I hold some or many men not to be so.

their reference upwards to the whole class: they hint at or tell of a proportion borne by the part to the whole. Some of them leave that proportion quite uncertain; others describe it vaguely; and others specificate till they reach numerical determination.

The logical "some" utterly ignores such reference. This is plain from the explanations already given. If the number of objects in a class were exactly ascertainable, our "some" would be broad enough to cover all of them but one, and narrow enough to admit one and keep out all the rest.[1]

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[1] Whenever particularity is carried, though it were but by a single step, beyond the negation of totality ("not-all"), we have passed out of the sphere of logic into that of arithmetic or mathematics. Number, it is true, is a logical præcognitum. But the positive ideas which logic postulates under it are only unity, plurality, and totality; and, specially, it postulates plurality only as being in thought the necessary link between unity and totality. It does not seek to develope plurality, positively, into any of those indefinitely various specifications which the repetition of the unit makes possible. When logic does aim at such development (and some very able logicians have tortured it into the task), it attempts what it cannot and need not do. It sets about performing, clumsily and imperfectly, a duty which the appropriate sciences of number and quantity execute with promptness and perfection.

So long, indeed, as the proportional specifications of particularity remain very vague, arithmetic and algebra give assistance so slight, that problems of the sort, though insoluble by pure or universal logic, are fairly and conveniently assignable to logic mixed or applied. Many such problems fall directly, or may easily be brought, within the scope of the rules given by logicians for modal propositions. Others are so easily dealt with as to require no rules beyond those of common sense. The premise "Most of the X's are Y's," evidently allows a wider inference than the premise "A few of the X's are Y's." As soon as we move on beyond such a point

(3.) Of the ordinary signs, some are definite, others quite indefinite, in their reference downwards towards the objects constituting the class. "Certain men (*quidam homines*)," is an example of the first kind; "Some men or other (*aliqui homines*)," belongs to the second.

The logical "some" is totally indeterminate in its reference to the constitutive objects. It is always "*aliqui*," never "*quidam*": it designates some objects or other of the class, not some certain objects definitely pointed out.[1]

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as this, we are, if we insist on continuing to use logical forms, doing really nothing more than throwing into logical forms results which we have gained by previous calculations, arithmetical or algebraical. This is true even of the simplest and most ingenious of all the devices of the mathematico-logicians;—Mr De Morgan's principle, called by Sir W. Hamilton the "ultra-total quantification of the middle term." The principle is this: that a half, and anything more than a half, are together in excess of the whole; and it yields a formula which merely saves us the trouble of working a simple equation, having oftenest an indeterminate solution.

[1] This third point, though implied in several of the received logical rules, has sometimes been overlooked. Surely it was so by those of the old logicians, who gave "*quidam*" as the logical sign. One or two of the Germans complicate the theory of predication needlessly, by admitting both readings.

Compare these two propositions: "Some X's are Y's;" "Some X's are not Y's." The popular "some," when unqualified, is naturally understood as indefinite; therefore common sense would lead us to say that, for all we know, both propositions *may* be true, but that the one or the other of them *must* be true. Logic, understanding the quantity of both X's as limited indefinitely, gives the same verdict. But sense and science would agree in granting a new trial, if the subject were, both times, "some certain X's." We should then have to call for evidence showing, whether the X's selected in the first proposition are the same X's which are selected

Both the second rule and the third seem to grow out of a consideration which may be explained thus. By making our quantitative limitation definite in either direction, upwards or downwards, we should really have thought out a new class, constituted by so many of the objects as we had thought of or named. The common attribute of the class would be the fact that the objects are so specified by us. And all so specified by us being signified by the term (say, "some certain X's"), this term would really, paradoxical as it may appear, be a common term distributed. It would be equivalent to "all those X's I am thinking of."

Definiteness, in short, is the distinctive characteristic of universality or distribution; indefiniteness is that of particularity or non-distribution.[1]

Development of the extension of common terms.

30. We must now treat, more closely than before, both of the relations, the objective and the attributive, which together constitute the totality of the concept, and of its sign the common term. Out of these will emerge by degrees one logical doctrine after another, till they yield at last their highest results in the theory of the syllogism.

The reference made by a common term to the objects thought as contained in the class, is called the *Extension*, Sphere, or Compass of the term: or, otherwise, the extension, sphere, or compass of a common term may be said to be

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in the second; or whether the two subjects designate two different ets of X's. If the sets are the same, one of the propositions must be false. If the sets are different, neither the truth nor the falsehood of the one would entitle us to infer either the falsehood or the truth of the other.

[1] Consult, as to all the quantitative signs, Hamilton's *Discussions*, the Logical Appendix.

constituted by all the objects thought as contained in the class.

In the broadest view, therefore, the extension of a common term is constituted by all the individual objects; and in any more limited view we can take, this ultimate reference to the individuals is silently implied. But the affirmation of extension by an enumeration of individuals, would be seldom (if ever) possible, and always useless. Every common term presupposes, in one view or another, several or many steps of generalization. Thus it has under it other common terms denoting contained classes of objects; while each of these may have other common terms under it; and so on, it may be, through many stages. When, therefore, common terms only are compared in respect of extension, the Extension of a Common Term is said to be constituted by all those other common terms, which are the names of classes or kinds of objects thought as included in the class denoted by it. Thus, one common term may have its extension constituted directly by several other common terms; each of which, again, has its extension constituted directly by several others: and, of course, the extension of the first covers all the extensions of all the others.

Concepts or common terms may be said to be *Ordinated in Extension*, when they are arranged in an order corresponding to the steps of generalization or specification. Ordination is most conveniently made from highest to lowest,—that is, from the one widest class, which contains all the others, down to the one or more narrowest classes, in which the data do not allow us to include any others. In respect of extension, we descend in the order of specification. The highest or most extensive term in such a series is said to be Superordinate to all the others; terms yielded by one and the same

step of generalization or specification are Co-ordinate; every term lower than the highest is Subordinate to all terms whose extension is greater, while it is superordinate to all, if there are any, whose extension is less. There are used also, as descriptive of ordination in extension (not in comprehension), the names Subalternant and Subalternate; to which there should be, and sometimes is, added, the name Co-alternate.[1]

It must be noted very particularly (though the point was observed before), that, as we descend in extension we are, at every step, thinking *away* objects, but thinking *in* additional attributes; that, as we ascend in extension, we are thinking *in* objects, but thinking *away* attributes.[2]

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[1] Thus, let us, assuming terms whose meaning and relations are simple and obvious, start from the class "organized beings" as a superordinate. One step lower in specification gives us, as the two classes constituting that class, "animals" and "vegetables," which are therefore subordinate to the first class, co-ordinate with each other, superordinate to any kind we may place under either. If, neglecting the class "vegetables," we descend in a loose specification with the term "animals," it might give us the six classes, "men, beasts, birds, fishes, reptiles, insects;" and these classes would be, all of them, subordinate directly to "animal," indirectly to "organized being;" they would be co-ordinates of each other; and, if our specification stop here, they would have no subordinates.

[2] Thus, our example sets out, in descent, from an indeterminate but large number of beings thought of as possessing the one attribute of "organization." At our next step, whether we regard the one term or the other in the co-ordination, we have a class containing fewer objects. For "animal" and "vegetable" together are required for including all "organized beings;" and each of the two classes wants all the "organized beings" contained in the other. But, contrariwise, whichever of the two subordinate classes we

31. The reference made by a common term to the attribute (simple or complex) thought as possessed by all the objects of the class, is called the *Comprehension*, Intension, or Content of the term: or, otherwise, the intension, comprehension, or content of a common term may be said to be constituted by the attribute (simple or complex) thought as possessed by all the objects.

Development of the comprehension of common terms.

If a term, given to have its comprehension evolved, presupposes but one step of generalization, the attribute is simple, or one; and no further evolution is possible than that which is yielded by the immediate import of the name. But each additional step of generalization gives an additional element to the attribute, which thus becomes complex; and each of these steps yields a new common term, the statement of which is a step in the evolution of the comprehension of the given term. Common terms which thus evolve the comprehension of a given common term, may be said

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contemplate, we see that its constitutive objects, though fewer than all "organized beings," possess an attribute which is *not* possessed by *all* organized beings, and is not the attribute on the thought of which the class was founded. "Animals" have the special attributes constituting "animal life;" vegetables have the special attributes constituting "vegetable life." It is needless to carry the analysis through the third stage.

The character of the ordination might be perceived from a different point of view, if we were to substitute, for each of the terms, its contradictory. "If negations are joined in thought to two concepts relatively higher and lower, there arises thus a reversal of their subordination. For, through the concept which contains the negation of a species, more objects may be thought than through those concepts which make up the negation of the genus." (Schulze, *Grundsätze der Allgemeinen Logik*, 1831, p. 54).

to be terms signifying attributes which are implied in the attribute given; or they signify attributes in respect of which the objects constituting the given class may be thought of as being included also in some other class or classes.

Cases yielding no possibility of evolution being excluded as barren, the Comprehension of a Common Term is said to be constituted by those common terms, which are thus significant of implied attributes. These are often, with suggestive propriety, called the Marks of the given term or concept.

Suppose, now, that there is given to us a series of terms ordinated in extension; and that we are called on to find among these the terms which are the intensive or attributive marks of some one term of the series. In what direction shall we look?—upwards or downwards?

The attributive marks must, all of them, be possessed by all the objects of that class of which the term whose comprehension is sought is a name. Consequently, the marks of the term will not be found in any one of the terms lower or less extensive than it is; for each of these lower terms signifies some attribute, possessed, indeed, by some objects of the class we start from, but wanting to others.

The marks of a term must be sought among the terms more extensive than itself. The objects of the given class possess the attribute signified by the term whose marks we seek: they possess also some other attribute, which is possessed by other objects besides; that is, they possess also some attribute in respect of which the given objects and these others are included in another class, which accordingly is more extensive than the first. The name of this more extensive class is a mark of the given class, in so far

as it signifies an attribute possessed by all the objects contained in that class.[1]

Common terms may be *Ordinated in Comprehension* as well as in extension; and it appears plainly, that the order in the one case must be exactly the reverse of that in the other. If arranged from highest to lowest, they will now stand in the order of steps in a presupposed generalization.

Again, to a series ordinated in comprehension there may be applied the same set of comparative names which were applied in the former ordination:—Superordinate, Co-ordinate, Subordinate. But, while co-ordinates continue to hold the same place, the terms which before were superordinate have now become subordinate, and contrariwise.[2]

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[1] Thus, the series of the last section does not yield any mark of the term "organized being;" no one of the lower classes signifies any attribute possessed by all organized beings. But, as a mark either of the term "animal," or of the term "vegetable," we may assign "organization," the attribute signified by the term "organized being;" and, as a mark of "man," or any of its co-ordinates, we may assign "animal life" as a mark in the first degree, and "organization" as a mark in the second. All animals may be marked as organized beings; all men may be marked as animals and organized beings.

[2] The terms of the example in the last section would have stood thus when ordinated in extension from above:—

1. Organized being; 2. Animal + vegetable; 3. Man + beast + bird + fish + reptile + insect.

The same terms ordinated from above in comprehension would stand thus:—

1. Man + beast + bird + fish + reptile + insect; 2. Animal + vegetable; 3. Organized being.

When explicit ordination is required as an aid for analysis, it is safest and easiest to make it in the descending order of Extension. The counter-relations are discoverable at a glance.

It is manifest, likewise, that, terms being given as expressly ordinated, either in extension or in comprehension, the other ordination is given by implication.

The law of concepts and common terms.

32. The results now gained enable us to generalize the distinctive law of concepts and common terms.

Extension and comprehension stand towards each other in an inverse ratio. By how much the more (or fewer) objects a class is thought as containing, by so much the fewer (or more) attributes are the objects thought as possessing: by how much the more (or fewer) the attributes are, by so much the fewer (or more) are the objects.

For predication through common terms, this is the universal Quantitative Law. Such predication, governed primarily and qualitatively by the principle of non-contradiction, is governed secondarily, and in the way of quantitative restriction, by this law of inverse determination.

All such predication may correctly be said to be nothing more than an explication, in the form of judgments and propositions, of those relations between the terms, which are implied in a pre-formed ordination. The same assertion may be made as to inference: for inference is merely a series of predications or propositions, in which implied relations are successively and systematically evolved.

The abstractive separation of the two wholes of the concept.

33. There is one limitation, narrowing our use of the determinative law of concepts when we come to use it in predication. To this limitation our attention cannot be too early called.

The concept, that which a common term signifies, is thought as a whole, whose parts also are thinkable. It has parts both of extension and of comprehension: it has parts

when it is considered in its relation to objects; it has parts when it is considered in its relation to attributes. Its totality is constituted by both kinds of parts taken together, not by either kind independently of the other.

If we are to think the concept, that is, the whole of signification of a common term, without any attempt at evolving the parts of either kind, we may and must think it as a whole whose constitution implies parts of both kinds.

But, if we desire to find the parts, or any of them—to determine what the parts are, and to think them, or any of them, distinctly—we cannot do so in both relations at once. We must seek, either to evolve the parts of the extension of the common term, leaving the comprehension unevolved as given, or to evolve the parts of the comprehension, leaving the extension unevolved as given. The complete evolution of the signification of a common term X, is a task to be performed only by the working of two problems, which must be solved separately. We must evolve the sphere, or extension, by determining what are all the kinds of X's, while we take for granted the attributes common to all the things so called; and we must evolve the comprehension, by determining what are the attributes common to all X's, while we take for granted the compass of the objects which bear the name. We must either think explicitly in extension, while we imply comprehension; or think explicitly in comprehension, while we imply extension.

While, therefore, a concept is one whole, yet, in reference to the possibility of abstractive analysis, its totality may be, and by logicians frequently and conveniently is, said to be constituted by Two Wholes. A concept, or its sign the common term, is evolvable so far only as it may be regarded as involving, on the one hand, a whole of extension

constituted by objects, and, on the other hand, a whole of comprehension constituted by attributes.*

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* The whole of extension has often been called the "logical whole;" and the whole of comprehension has, by some of those logicians who have generalized its laws, been said to possess the character assigned, by others than logicians, to a "metaphysical whole." The names point to a distinction worth noting.

Yet the warning must be added, that they will deceive if they tempt us to infer the exclusion of comprehension from logical scrutiny. The warning is the more needed, because this aspect of the concept is by far the more difficult of the two, both for thought and for expression; and because in our logical systems the weakest point is the development of it.

So much of doctrine will hereafter be founded on the correlation of extension and comprehension, that it may be well, once for all, to bespeak close attention to the principle, and to notice generally, at the cost of a little anticipation, the historical position which the doctrine holds in the science.

Neither of the two relations of the concept could be, or ever has been, altogether overlooked. But extension, which always predominates in thought, and thus modifies all natural forms of speech, long usurped in the logical field a place almost so broad as to leave no room for comprehension. So it was with Aristotle. So, likewise, was it with the schoolmen, who held that the *totum universale*, the whole of extension, is that with which only logic has directly to do; and that the science cannot look further away from it than for seeking marks (*notæ*) by which the mutual relations of universal wholes and parts may be determined. Thus comprehension lay in the dark.

"The distinction," says Hamilton (*Discussions*, p. 641*), "as limited to the doctrine of single notions, was first signalized by the Port-Royal logicians, under the names of extension and comprehension. Leibnitz and his followers preferred the more antithetic titles of extension and intension (though intension be here somewhat deflected from its proper meaning, that of degree); and the

quantitas ambitus and *quantitas complexus* have, among sundry other synonymes, been employed in modern times—not exclusively, for Aristotle uses τὸ περιέχον and τὸ περιεχόμενον. The best expression, I think, for the distinction, is breadth (Πλάτος, *latitudo*), and depth (Βάθος, *profunditas*)." (See the Port-Royal *Logique*, part i., chap. vi.) Both the correlation and its law (of the inverse ratio), speedily became familiar to all logical students. "It is," says Reid, "an axiom in logic, that the more extensive any general term is, it is the less comprehensive; and, on the contrary, the more comprehensive, the less extensive." (Hamilton's *Reid*, p. 390.) But in our climate the doctrine bore no fruit.

In the *Logik* of Kant the correlation is alleged, and the law of the inverse ratio stated; the first steps also are taken towards those applications of the principle which speedily followed. Since then, it has been trite doctrine in the German schools, that a definition is a predication making distinct the comprehension of a concept; that a division is a predication making distinct the extension: and, while the German logicians have not all generalized with equal clearness the law of the inverse ratio, their success in expiscating the theory of both processes has been proportional to the clearness with which they have apprehended the principle. Compare, for instance, Twesten with Fries. At this point, however, the German logicians have come to a stand. With a solitary exception, none of them, so far as we know, suspected, till very lately, the possibility of bringing the double relation of the concept to bear on the syllogism. The one exception was Beneke; who, however, after having seemingly grasped the principle very firmly, let it slip out of his hands before it had yielded any generalized doctrine. (See his *Lehrbuch der Logik*, 1832, chap. viii., §§ 170, 171, 182.) It should be added that Beneke saw, very clearly, how the distribution of the predicate in affirmative propositions is necessary for the consistent development of the relation of comprehension. (See his sect. 182, foot of page 124.)

This one hesitating anticipation required in fairness to be noticed. But it leaves untouched the essential originality, as well as the whole value, of Sir William Hamilton's masterly application

of the counter-wholes to the elucidation and firmer grounding of the theory of the syllogism. This deepest section of his logical system seems to have been as yet little studied. But it may not be presumptuous to hint a belief, that his formal doctrine of the thorough-going quantification of the predicate will be found to have its chief value, and perhaps its only practical applicability, in its efficiency as an instrument for evolving the syllogistic bearings of the comprehension and extension of concepts. Some of these bearings it will be attempted by and by to explain.

Yet, further, it has to be remarked that, contemporaneously with Hamilton, two other great logicians have seized the same thought; both of them, however, grasping it from the negative side, and not working it out to any positive formal results. Professor Trendelenburg lays it down in the broadest terms, that the law of the syllogism can be understood only through the mutual relation of extension and comprehension. (*Logische Untersuchungen*, 1840, ii., pp. 232–250, § 16.) Mr Mill has a much less firm hold of the idea, mainly by reason of his avoidance of the formal point of view; but in the counter-relation of the two wholes lies the clue to the distinction which he has used so skilfully in working out his own system—that between the denotation and the connotation of terms.

PART SECOND.

THE DOCTRINE OF PROPOSITIONS.

CHAPTER I.

The Forms of Categorical Predication.

The character of categorical predication.

34. The only kind of proposition which is the direct expression of a simple judgment, is that which is technically called the *Categorical*. From those other kinds which are usually compared with it by logicians, it may be distinguished with sufficient exactness, when it is described as being an assertion or predication, affirmative or negative, not limited either by a condition or by an alternative.[1]

All categorical propositions are formally resolvable, though not all with equal ease, into three parts or factors:—The *Subject*, which is a name for that which is spoken of; the *Predicate*, which is a name for that which is said of that which is spoken of; the *Copula*, a verb, in which the assertion is expressed, and which likewise qualifies the assertion as an affirmation or a denial. The subject and predicate are called the *Terms* of the proposition.

[1] "X is Y" is a categorical proposition. Examples of the other kinds are these:—Of the Hypothetical, "If X is Y, it is Z;" of the Disjunctive, "X is either Y or Z."

In many common forms of speech, the copula is mixed up with the predicate: but they may always be separated; and for exact logical analysis they must be so. The pure copula is always "is" or "is not," "are" or "are not;" words which, when discharging this function, do not import existence, nor even any mode of time, but merely the fact that the things thought as denoted severally by the subject and by the predicate, are thought in relation to each other.[1]

Propositions qualitatively resolvable into assertions of identity or difference.

35. The *Quality* of a proposition is the character of the predication it contains. As being the expression of a judgment, predication can have only the one or the other of two characters, and cannot have neither. It must be either Affirmative or Negative. The copula, which expresses the act of predication, must either want, or have, the negative sign "not."

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[1] The assertion, "The world—is," passes readily into the explicit form, "The world—is—something that exists." Many other resolutions are equally easy. "John thinks," becomes "John—is—a person thinking." The first in each of these pairs of propositions would have been called, by the old logicians, a *propositio secundi adjecti* (or *adjacentis*), as having but one factor expressly adjected to the subject; the latter would be a *propositio tertii adjecti* (or *adjacentis*), as having a second factor also expressed.

The infinitive mood is a substantive, and is most easily useable in its gerundive form; and cases where it is one of the terms are those that oftenest present the predicate before the subject. "It is pleasant to know = All knowing is a thing pleasant." The propositions which the Germans have called existential, expressible by impersonal verbs (as "it thunders, it rains"), may always be regarded as expressions of an incomplete cognition, of a thought which we either cannot analyse or have not taken the trouble to attempt analysing.

The question of quality emerges in regard to every proposition. It is the primary question, and also the most important of all.

The doctrine to be kept steadily in view is that which has already been laid down, and in part illustrated. In the data of every proposition, there is an hypothetical presupposition of duality: two ideas are given, whose designative terms are available as subject and predicate. The proposition intimates whether, in respect of the relation under which the objects are thought, the duality can or cannot be reduced to unity. The terms having been compared, the proposition expresses the determination of the thinker on this question: whether the object or objects denoted by the subject be identical or non-identical with the object or objects denoted by the predicate. An affirmative proposition predicates the identity of subject and predicate; it does so in all circumstances. The objects are asserted to be the same objects; although, when regarded from one point of view, they bear the name given them in the subject, and, from another point of view, the name (if it be a different name) given them in the predicate. So a negative proposition predicates in all circumstances non-identity or difference; it asserts that, whatever may be the names, or whatever the points of view from which the objects are regarded, the one object, or group of objects, is a different object or group from the other.

The question of identity or difference is the main question as to all objects compared in judgment; as to certain kinds of objects it is the only question.

36. When both subject and predicate are singular terms,

Predication through singular terms.

the quality of the proposition is the only point to be considered. Especially, there can be no question as to the quantity of the terms, each of which must signify an individual object, and cannot admit any limitation to its meaning. The proposition is a pure predication of identity or non-identity. Of the individual designated by the subject there may be affirmed, as predicate, any term which is merely another name for the same individual. Of that individual there must be denied, as predicate, any and every term which is a name for any other individual.

Cases of either sort arise too infrequently, and, when they do arise, are too easily disposed of, to require special rules.

The quantity of common terms.

37. The question of *Quantity* arises when common terms enter into propositions, as subject, or as predicate, or as both. Common terms being capable of signifying either all the objects of a class or less-than-all of them, the question of quantity relates to the terms. Are they distributed or undistributed?

The answer to this question as to the terms, serves only to guard and limit the affirmation or negation made by the copula. Rigidly and rightly considered, the determination of the quantity of a term, whether through its known meaning, or through interpretation of its sign, is nothing more than a method of protection against that ambiguity, which besets common terms on account of their capability of denoting either all or less-than-all. That which is properly a term in a proposition (whether subject or predicate), is not the common term in its capability of quantitative signification, but the common term as definitely interpreted to mean all or some.

This interpretation being gained, we proceed, when both

terms are common, to decide whether the objects denoted by our subject (all, or some, of the objects constituting a certain class), are, or are not, the very same objects denoted by our predicate (all, or some, of the objects constituting a certain other class).

In a word, the determination of the quantity of the terms in a proposition, is nothing else or more than a step of preparation, in cases requiring it, for the determination between identity and non-identity, and for the consequent choice between affirmation and negation.[1]

The four received forms of predication through common terms.

38. In regard to quantity, the received logical doctrine and nomenclature may be set down as follows:—

Propositions whose subjects are common terms, are said to have quantity, that is, variable quantity. A proposition

[1] "All metals are minerals = All metals—are—some minerals." The terms might be held to be the two class-names "metals," "minerals;" and in this view the proposition might be described as being an assertion of partial identity between the two classes. "The whole class metals—is identical with—some part or other of the class minerals." But such an analysis is apt to cause indistinctness of thought, and that because it does not go far enough. Our terms are properly not the class-names taken without fixing of quantity, but these names as quantitatively determined by the signs; in other words, our terms here are names positively used to designate all the objects of the first class—some or other of the objects of the second. Thus regarded, the proposition is seen to be an assertion of total identity, between the objects denoted by the first name and the objects denoted by the second. "The objects which, in respect of certain properties possessed by them, and not possessed by any other objects, I call 'all metals'—are (or are the same objects with)—the objects which, in respect of certain properties possessed by them, but possessed also by other objects, I call 'some minerals.'"

is said to be Universal in respect of quantity, when its subject is distributed; it is called Particular when its subject is undistributed. The quantitative sign of the subject, "all," "any," or "some," if not given, is to be supplied. A proposition whose subject is a singular term cannot receive a sign, but must be treated as a universal.

It is admitted, by all logicians, that the predicate, when it is a common term, must, like the subject, have its quantity positively determined: it must, in every proposition, be either distributed or undistributed. Ordinary language, however, does not indicate the quantity of the predicate by any prefixed signs; and, in the received logical systems, no sign is supplied. It is held that the necessity for one is superseded by a fixed rule of interpretation. The quantity of the predicate, we are told, is fixed by the quality of the proposition, without any regard to its quantity: the predicate is distributed in all negative propositions, whether universal or particular; it is undistributed in all affirmatives. The reasons assigned for the rule are these.—A negative proposition cannot but distribute the predicate; for when, of anything whatever denoted by the subject, we deny the class denoted by the predicate, we deny that the subject is to be found anywhere in the predicate-class, or makes any part of it; or we affirm, in effect, that the subject is excluded from the whole of the predicate. On the other hand, it is allowed, that an affirmative proposition either may or may not distribute the predicate. When, of anything whatever denoted by the subject, we affirm the class denoted by the predicate, we may mean, either that the subject constitutes the whole class, or only that the subject is contained in the class, or is a part of it. In the first case the predicate is distributed; in the second it is not so. But, it is alleged,

the latter case of the two is the only one with which logic can deal. The narrowest meaning which an affirmative can bear, is the assertion that the subject is a part of the predicate; so much, therefore, may always be safely assumed. If the signification of the proposition really does embrace the wider alternative, the fact is discoverable only by means lying beyond the sphere of logic, a purely formal science, which possesses no machinery for interpreting the terms, or for otherwise working on the matter of propositions.[1]

Accordingly, the common scheme of propositions, and the scheme of inferences founded on it, are confined to forms of predication from which affirmatives that distribute the predicate, and negatives that do not, are alike excluded.

This exclusion being made, the possible Forms of Predication, through common terms, are necessarily no more than four. For the sake of brevity in naming, those four kinds of propositions are noted by the first four vowels. The letter A denotes a universal affirmative (subject distributed, predicate undistributed); I denotes a particular affirmative (subject undistributed, predicate undistributed); E denotes a

[1] It is not to be wondered at, that the peremptory refusal to look at the meaning of the terms should be adhered to by the German logicians; among so many of whom, since the time of Kant, the purely formal or *a priori* character of logical science has been a cardinal article of faith. Yet, in not a few of the German systems of logic, the doctrine of definitions and divisions (which are admitted to be universal affirmatives distributing the predicate), is very thoroughly expounded, the case being treated as exceptional. It might surprise us more that the refusal should be insisted on so generally among English logicians; since by them the exclusion of matter from logical scrutiny has, though usually asserted as a rule, been scarcely ever traced up to principles, while it has been practically departed from in many other points of doctrine.

universal negative (subject distributed, predicate distributed); O denotes a particular negative (subject undistributed, predicate distributed).

The designation of propositions as universal or particular, in respect of the quantity of the subject, cannot be questioned. The four forms marked by the vowels are likewise unchallengeable.

But it has lately been questioned whether logic is either bound, or so much as entitled, to exclude all forms of predication besides the A, I, E, O. Both of the exclusions have been condemned; not only the exclusion of affirmatives which distribute the predicate, but even the exclusion of negatives whose predicate is undistributed. These points, therefore, must be more closely examined.

It has likewise been proposed, that, in the preparation of propositions for logical treatment, the signs of quantity be prefixed to predicate as well as to subject. This express signature of the quantity of the predicate is fruitful in results, to an extent which would scarcely be anticipated from an expedient so simple and so purely formal. It will be adopted in our further progress, with all examples where exact analysis is aimed at.[1]

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[1] The express signature of predicates is a proposal of Sir William Hamilton's. Lambert, indeed (*Neues Organon*, 1764, p. 115), had invented a scheme of logical notation, in which effect was given to the quantity of every term; and Ploucquet (1761) had suggested the prefixing of the quantitative sign to the predicate in all assertions expressed for logical use. (See Fries, *System der Logik*, ed. 1837, p. 103.) But the signature of the predicate was still, by all later logicians, unadopted. Its effects are surprising. Doctrines already admitted and proved are, by means of it, made more easily explicable; other doctrines become traceable to principles which had hitherto been overlooked; and there are brought to the surface

39. Every proposition has two alternatives of quality. Every common term has two alternatives of quantity; and the terms of every proposition are two; consequently, if quality is to have no effect, every proposition has four alternatives of quantity. If, then, we look merely to the combinations of number, the possible forms of predication must be eight.

The eight possible forms of predication through common terms.

The following formulæ exemplify these eight kinds of propositions, note the quantities, and explicate the asserted identities and differences:—

1. A. All X's are some Y's = All the X's—are identical with—some or other of the Y's.
2. I. Some X's are some Y's = Some or other of the X's—are identical with—some or other of the Y's.
3. A^2. All X's are all Y's = All the X's—are identical with—all the Y's.
4. I^2. Some X's are all Y's = Some or other of the X's—are identical with—all the Y's.

5. E. Any X's are not any Y's = All the X's—are non-identical with—all the Y's.
6. O. Some X's are not any Y's = Some or other of the X's—are non-identical with—all the Y's.
7. ½E. Any X's are not some Y's = All the X's—are non-identical with—some or other of the Y's.
8. ½O. Some X's are not some Y's = Some or other of the X's—are non-identical with—some or other of the Y's.[1]

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new doctrines, which had been unsuspected because the quantity of the predicate, through its want of express marks, had not been attended to unless when it bore on questions already raised.

[1] *Examples in Significant Terms.*

1. A. All men are imperfect = All men—are—some beings imperfect.

To the received scheme of predication, there would thus be added four forms, two affirmatives and two negatives. All of these express possible forms of thought; and, accordingly, admission has been demanded for all of them, as necessary for the completion of that theory of thought which logic undertakes to set forth. Admission, again, has been demanded for the affirmatives, but refused to the negatives. The grounds of both claims require examination.[1]

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2. I. Some men are happy = Some men—are—some beings happy.
3. A^2. All men are responsible animals = All men—are—all animals responsible.
4. I^2. Some men are logicians = Some men—are—all logicians.

5. E. No men are stones = Any men—are not—any stones.
6. O. Some men are not wise = Some men—are not—any persons wise.
7. ½E. No men are some Z's = Any men—are not—some (or other) of the Z's.
8. ½O. Some men are not some Z's = Some men (or other)—are not—some (or other) of the Z's.

[1] The A^2 at least might have been expected to be acknowledged by some of the German logicians, who perceived so exactly the character of definitions and divisions, and were compelled to admit that these are instances of that form. But they content themselves with repeating the old declaration, that the distribution of the predicate in affirmatives, when it does occur, is "accidental"—"not cognizable from the position at which logic takes its stand." If the view be correct which will be stated in the text, it is more to be regretted that they had not given reception to I^2, especially since one of them has been quite aware that it is the full and only adequate converse of A. The observation is Beneke's, in a passage of his *Lehrbuch*, p. 182, which has already been referred to (note to section 33.) The only formal recognition of the distributed predicate in affirmatives, which we have observed among the German logicians,

40. More than one characteristic feature of predication may be thrown into light, if we consider every proposition as being, actually or possibly, the answer to a question. A

The six available forms of predication through common terms.

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is that of Hoffbauer, who not only recognizes reciprocal propositions (A^2), but lays down rules for syllogisms having both premises of that character. (*Anfangsgründe der Logik*, ed. 1810, pp. 97, 100, 185.)

In Mr George Bentham's *Outline of a New System of Logic* (1827, p. 133), all the eight possible forms are correctly set forth. But the writer instantly loses hold of the clue he had grasped: indeed, he goes so far astray as to maintain, that in negatives it is a matter of indifference whether the predicate be distributed or not. He ends by returning to the A, E, I, O. In Mr Solly's *Syllabus of Logic* (1839, p. 47), the eight forms are stated as arithmetically possible; and their character is shown by the prefixing of signs to the predicate. But the four added forms are at once thrown aside, as never introduced in practice. The claim for all the eight is made by Sir William Hamilton; and all of them are worked by him into his scheme of syllogisms. The admission of both quantities of predicate, with both qualities of copula, makes up his "Thorough-going quantification of the predicate." (See his *Discussions*, Appendix ii.; and Baynes' *New Analytic*.) Mr Thomson (*Laws of Thought*, 1842, 1849, 1854) rejects the additional negatives, but incorporates the additional affirmatives into his syllogistic tables. They are his U and Y. The symbols here proposed for those two affirmatives (A^2 and I^2) seem to have some advantages. While easily pronounceable, they intimate a relation of the two forms to the received A and I; and the character of this relation is faintly hinted at, when the added forms are symbolized as higher powers of the old ones. The propriety of the two negative symbols, $\frac{1}{2}E$ and $\frac{1}{2}O$, is a matter of very little consequence: they are to be thrown aside; and it is enough that we have brief modes of naming them in discussing the reasons for and against their reception.

In reference to Sir W. Hamilton's system, it may be well to remark, that his invaluable suggestion of always marking expressly the

problem is propounded in thought: a judgment is the solution. We ask, what is B? We answer, B is X. That which is denoted by the subject is always the ***datum;*** some-

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quantity of the predicate is one thing, that his proposed extension of the propositional forms by "thorough-going quantification" is another. In Mr Baynes' Appendix are interesting quotations from old logicians, who have contemplated the distribution of the predicate in affirmatives, and signified it by the universal sign. Instances, too, are cited, in which the bearing of this distribution on the syllogism is hinted at.

Perhaps the following passages are more decided in the application of the distributed predicate, than any of Mr Baynes' quotations. They carry us from the fourteenth century to the sixteenth; from an Englishman, the "prince of nominalists," one of the greatest of the earlier schoolmen, to a Scotsman, who has been called the last of the schoolmen, and was far from being the least subtle of the band.

In Occam's *Summa Totius Logicæ,* one chapter (lib. iii., cap. 13) is described in its title as showing "in what cases we may syllogize from two affirmative premises in the second figure." Two cases are described. The first is that in which the middle term is a singular. It is the second case that interests us here. "Secundus casus est, quando medius terminus sumitur cum signo universali. Tunc semper contingit inferre conclusionem affirmativam, in qua major extremitas prædicatur de minori. Bene enim sequitur: 'Omnis homo est omne risibile; Socrates est omne risibile; ergo Socrates est homo.'" "Iste autem discursus probatur per hoc: quod semper talis propositio major convertitur in unam universalem affirmativam; qua conversione facta, patet quod discursus est in prima figura, regulatus per dici de omni." The A^2 is here given twice. Only it is noticeable that the proposition is not considered, as it might have been, to be convertible into another A^2 (which would have yielded an unrecognized mood in the first figure); and hence it is that the predicate of the conclusion is undistributed. In the next paragraph the I^2 is, though not exemplified, unequivocally de-

thing expressible as a predicate is the *quæsitum.* The subject may be called the antecedent, the predicate the consequent; and the hypothetical or conditional form of stating a

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scribed; and it is correctly alleged to justify syllogisms with two particular premises. "Et est sciendum, quod in duobus prædictis casibus non solum contingit arguere ex universalibus affirmativis; sed etiam contingit arguere ex omnibus affirmativis particularibus; et eodem modo probantur tales syllogismi ex particularibus sicut ex universalibus. . . . Et tenet talis discursus, non gratia materiæ, sed gratia formæ: quia, in omni materia, observato quod medius terminus sit terminus discretus, vel sumptus cum signo universali in majori, discursus est bonus."

The other authority is Joannes Major (John Mair), now remembered only as an historian; who, besides teaching in Paris, was a regent, and afterwards provost, of the College of St Salvator in the Scottish University of Saint Andrews. His *Introductorium in Aristotelicam Dialecticen,* printed at Paris in 1527, while it shows close study of Occam's doctrines, is prominently marked by the writer's characteristic independence of thought. He makes very frequent use of the distributed predicate in affirmatives. The following points are especially noticeable.

1. Instead of rejecting the form, he merely says it is uncommon: "affirmativæ prædicatum raro distribuitur." (*fol.* cxlviii., *col. d.*) 2. In expounding the reciprocity of deduction and induction ("descensus" and "ascensus"), he insists on the universality of the predicate, in a collective acceptation of the sign, as a fixed datum; "constantia est hæc propositio; isti pomi sunt omnes pomi. . . . istæ arbores sunt omnes arbores." (*fol.* cxiii., *a, b.*) 3. He indorses Occam's verdict on the second figure, lays down a principle for protection against resulting fallacies, and assigns a practical reason for the limitations assumed in the received syllogistic rules. "Dices forte, hæc consequentia est bona: 'omnis homo omne animal est; et omnis asinus est animal: ergo omnis asinus est homo.' . . . Respondetur in uno verbo. Ubi a mendis, ratione quorum regulæ sunt traditæ, præcavetur, majore particulari aut utravis præmis-

proposition places the terms expressly in that relation :—" If B is B, it is X."

We have seen, already, that concepts are the only consequents yielding any positive knowledge worth having; that common terms are the only predicates yielding affirmative propositions worth expressing. Among common terms, then, our predicates are sought. We desire to affirm of our subject the name of a class. A known class will yield a predicate, if we can think that our subject makes even a part of it; and, if this is all we can think, our predicate will be undistributed. If our subject is a singular term, our affirmation cannot embrace the predicate more widely. The individual denoted by it can be only one or another of the objects which constitute the class indicated in our predicate.

But, if our subject is a common term, the objects it de-

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sarum affirmativa, discursus in hac figura, *sicut in aliis*, est formalis. . . . Primi regularum traditores de propositionibus communiter consuetis loquuti sunt; hoc est, de affirmativa cum prædicato distribubili non distributo, et de negativa cum prædicato distributo." (*fol.* clvi., *c.*) 4. Afterwards he deals similarly with the third figure. (*fol.* clviii, *b.*)

It may be noted, also, that both of those dialecticians treat, and Occam very diffusely, a current scholastic distinction which appears in one of Mr Baynes' quotations. The "suppositio" of terms in propositions (that is, their objective reference), was said to be of two kinds, "determinata" and "confusa;" the latter, again, being either "confusa tantum" or "confusa et distributa." The complexity and vacillation of the old rules of "suppositio" seem to have sprung from two sources: an indistinct apprehension of the effect which *non*-distribution of the predicate has on affirmation; and a frequent attempt to identify singular terms with common terms undistributed.

notes may be either some, or all, of the things constituting a class denoted by another common term; or, again, they may not be any of the things constituting that class. Thus there arise three cases, all of them possible, actual, and more or less frequent.

First, We may be entitled to think of the objects denoted by the subject as being only some, not more, or to think of them as being certainly some, though we do not know whether they are or are not all, of the things denotable by the predicate. Either state of our knowledge will yield an affirmative predication, having the character of the affirmatives in the received list. It will be an affirmative with an undistributed predicate, an A or an I, as the subject is distributed or undistributed. Such propositions may conveniently be called Propositions of Inclusion: they assert only that the subject is included in the class which yields the predicate.

Propositions of inclusion make up a very large majority of the affirmatives that actually occur; and perhaps they are, without exception, the only affirmatives which we ever use exhaustively as data, unless when, as in scientific discussions, we reason from definitions.

Secondly, however, there do occur also affirmatives which may be called Propositions of Constitution. In these, the things denoted by the subject are thought of as being all the things denotable by the predicate: they are asserted to constitute the class of which the predicate is a name. The propositions are affirmatives with distributed predicates. Universal propositions of this type, the A^2 of our formulæ, are exemplified by definitions, and also by logical divisions. Particular propositions of the kind, the I^2 of our formulæ—particular propositions which are interpreted imperfectly unless the predicate is held to be distributed—occur more

frequently than we are apt to suppose. We shall encounter them, by and by, as being really the only complete and direct converses of the A of the received scheme.

Propositions really treatable as A^2 and I^2, have been currently handled by logicians, and are very frequent in ordinary thought. They are technically spoken of as exclusive propositions: "Men are the only responsible animals." They are usually treated as compound. The example is resolved into these two assertions:—1. "Men are responsible animals;" that is, they are some at least of the class; the question, whether they are the whole class, being supposed to be in the first instance undecided: 2. "Creatures which are not responsible animals are not men." But, if we allow distribution of the predicate, the proposition is interpretable as expressing one simple judgment: "All men—are—all responsible animals."[1]

[1] For the only uses to which it is here intended to apply either A^2 or I^2, it is scarcely necessary to raise a question, which, however, would require consideration if these forms were to be worked up into additional syllogistic moods. The "all" of the received A is distributive. Can it be so in these added forms? or, is it necessarily collective? Sir William Hamilton declares incidentally that the totality may be thought either way. "We can say, as we think, affirmatively, 'All triangles are all trilaterals:' This collectively, 'the whole (or class) triangle is the whole (or class) trilateral:' this distributively, 'every (or each several) triangle is every (or each several) trilateral.'" (*Discussions*, Appendix ii., p. 627.) It is difficult to see one's way clearly through the distributive interpretation. Perhaps it may be justified thus:—Let the given proposition be, "All X's are all Y's." Collectively taken, the assertion is, that the aggregate of the X's is the same with the aggregate of the Y's; that is, the whole class X is the same with the whole class Y. Distributively taken, it may be regarded through the names:—

When, therefore, the purpose is to predicate a relation between the subject and a class, there are data for affirmation, first, when we are able to assert inclusion; secondly, when we are able to assert constitution.

Thirdly, The same purpose being entertained, we have data for negation, when we are able to assert exclusion. A Proposition of Exclusion is one which asserts that the things denoted by the subject are excluded from the class denoted by the predicate; that, in other words, they do not make any part of that class, or that they are non-identical with all the things which that class contains. Evidently such propositions have the predicate distributed. They are the E and O of the received scheme.

They are not only of continual occurrence, but widely useful. In every kind of inquiry, we are able to deny a great deal more than we are able to affirm; and a denial which entitles us to set aside a whole class of things as being not the things we are interested in, is often one of the most valuable of all steps towards our learning what the things we investigate positively are.

The two non-available forms of predication through common terms.

41. When we desire to explicate our implied knowledge by referring our objects to a class, the three judgments expressible by the three kinds of propositions which have now been explained, appear to be all the judgments that can either constitute positive knowledge, or be steps leading towards it. We must assert either inclusion, by A or I; or constitution, by A^2 or I^2; or total exclusion, by E or O.

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each of the things which, when viewed from a certain point, we call X, would, when viewed from another point, be each of the things we call Y.

Propositions having the character of our seventh and eighth formulæ, $\frac{1}{2}$E and $\frac{1}{2}$O, do not seem to occur at all. Can there be detected, in actual thought, any examples of negatives, whose predicate, when its true function is brought to light, proves to be undistributed? One should not expect to find such. They could not serve any conceivable use, either as data for inference, or as conclusions to be inferred. We know, or are on the way towards knowing, when we are able to assert, either that our subject is *in* a class, or that it *constitutes* a class, or that it is *out of* a class. But the propositions in question do not assert any of these three things. They assert, not knowledge, but doubt: and the doubt which they do assert does not cover any the tiniest germ of knowledge, in regard to the objects from which we started,—those which are denoted by the subject, and which we wish to determine, positively or negatively, through the predicate.

In the formation of an opinion in regard to them, the indefinite character of logical particularity must be kept sternly in view. If the logical "some" were definite, those negatives would be virtually the received E and O. They would assert the exclusion of the subject, not, indeed, from the whole class denoted by the term which is formally the predicate, but from a certain fixed part of that class, which part would really be a sub-class, and ought to yield a name which would be the true predicate. But the logical "some" is, and must be, indefinite; and it is on this footing that the propositions have been placed, when they are asserted to be forms of thought, the analysis of which ought to have a place in logic.

Whenever the subject of a proposition is indeterminate in quantity, because particular (as "some or other of the

X's"), we have a very narrow field both for predication and for inference. The defect, however, is often unavoidable. The subject is our datum; it is the name of that which is given us to be judged of. But, be our subject quantitatively determinate or not ("all" or "some"), we seek for it a predicate which shall force us to assert, on pain of self-contradiction, the identity or non-identity of the objects denoted by the subject with objects denoted by the predicate. A predicate quantitatively undetermined by being particular, will yield an affirmation, A or I. In judging that "The X's (all or some) are some or other of the Y's," we have found for our subject a positive place in the field of our knowledge,—a place somewhere among the objects we call Y's: we have identified our subject with some or other of the Y's; and we are put on the track towards discovering its place still more exactly, through subsequent scrutiny of the Y's. But, if we must judge negatively, an undistributed predicate does not fix the place of our subject anywhere, either among or not among the objects we already know. The assertion that the X's (all or some) are not some or other of the Y's, does not contradict either the assertion that our given X's are things different from all the Y's, or the assertion that they are identical with some things or other lying in those parts of the sphere of the Y's, which our given predicate must have left unfilled. In a word, our proposition is nothing better than an involved expression for a barren alternative. Our X's, we learn, either are Y's, or they are not Y's; which is no more than what we know to be true, by the axiom of determination, in regard to any term whatever in its relation to any other.

It is well, then, that propositions having this character

should be recognized as expressing possible forms of thought; it is well that we should know every garb, in which even doubt and ignorance may clothe themselves. But there does not appear to be any sufficient reason for complicating the rules either of inference or of predication, by extending them to forms which yield no real explication of any given thought. Certainly such a proposition is never given to be inferred from. If such a proposition is the only one that can be inferred from another, the fact is a significant testimony to the poverty of the datum.[1]

The special uses of propositions of constitution.

42. Our scrutiny of the eight possible forms of predication leads to this result.

The four forms of the schools retain their place without challenge from any quarter; and doctrines bearing on them must always constitute the main part in the logical theory both of predication and of inference. Our chief duty must be the development of them: of the two affirmatives, A and I, propositions of inclusion; of the two negatives, E and O, propositions of exclusion.

But there do not seem to be good reasons for absolutely refusing a place in the logical system to propositions of constitution—the affirmatives which have been marked as A^2 and I^2. When an affirmation in which the predicate is distributed occurs in actual reasoning, we cannot apply to it rules which suppose its predicate to be undistributed, without the risk of either contracting unduly the limits of inference from it, or admitting wrongly the validity of inferences through which it may have been gained.

[1] See Note First at the end of the chapter.

The latter of the dangers is probably the more imminent of the two. The increase in the power of inference through distribution of the predicate proves, on narrow inspection, to be by no means so large as we might expect it to be. Besides this, neither of the added forms can actually occur in reasoning, as data or premises, unless in the way of exception, and in circumstances making it easy for any one familiar with logical principles to apply the necessary correction to the conclusion. At all events, no attempt will here be made to work these forms, as premises, into the received scheme of the categorical syllogism.

But they should and will be used, as materials of great value for fortifying some weak points of the current logical system. Definitions and divisions cannot be thoroughly understood, unless through the A^2. Disjunctive propositions rest wholly on it. The I^2, again, is imperatively required for giving consistency and completeness to the theory of conversion; and through this process it has bearings on the syllogism.

If the current objection is urged, that the mere form of an affirmative does not enable us to know whether the predicate actually is distributed or not, the answer is not far to seek. It is true that we are not, *qua* logicians, able to interpret our terms, far less to decide the question of truth or falsehood for any one proposition considered by itself. But, even when one proposition only is given, we are entitled, before we undertake dealing with it, to demand, from without, all the information required for enabling us to apply logical laws.

The information we do demand is not extensive. It is wholly embraced in the two postulates laid down in our preliminary inquiries. We ask to be informed, in regard

to every term given, whether it is to be understood as a singular or as a common term. If it has the latter meaning, we ask to be informed whether it is distributed or undistributed. For negatives, the information comes of itself. For affirmatives, we are entitled to summon it. If such a proposition is really either an A^2 or an I^2, we have a right to require warning of the fact. If, on the other hand, we evolve either form for ourselves, we are equally well entitled to make the peculiarity clear, by prefixing of the quantitative sign to the predicate.

The interpretation of propositions.

43. All predication is reducible into categorical forms; and, as it has already been alleged, every categorical predication may be dissected into the three constitutive factors of the proposition,—Subject, Predicate, and Copula. A proposition is not naturalized in our realm,—it has neither acquired logical privileges, nor become fully amenable to logical laws,—until it has submitted itself to both steps of this transformation, and has completed its legitimation by obedience to the postulates.

Such a process of preparation, while it lies beyond the function of pure logic, pre-supposes, likewise, interpretation of the terms; a duty which is still more distant from ours, and which can seldom be performed efficiently without a scrutiny, utterly extra-logical, of the truth or falsehood of the given assertions.

But, in a system of applied logic, an introductory section might, fitly and advantageously, be employed in such an analysis of the ordinary forms of predication, as should exhibit their relations to the logical forms, and found rules or aids both for interpretation and for transformation. Even for the design here entertained, some such assistance may

advisably be offered; although it cannot, and need not, embrace any modes of expression except a few of those which are likely to prove most troublesome in elementary logical study. Assertions made for purposes other than logical, do seldom wear a shape fitting them for logical use; and we may warrantably turn aside for a little, to examine some of the most common varieties of predication, and to discover, if we can, in what way, and how far, they may be made available as elements of inference. A few hints to this effect are thrown into the second of the notes appended to the present chapter.

Note I.

Sir W. Hamilton's Partial Negatives.

Our seventh and eighth propositions, marked as ½E and ½O, are the new and peculiar forms of predication in Sir William Hamilton's system: they are his Partial Negatives, Toto-partial and Parti-partial. Forms so authoritatively recommended cannot be so much as questioned, without a painful distrust in one's own judgment; nor can they be set aside but with reluctance and hesitation, even if the ground of dissent should seem to be very firm. It is right that the argument should be stated more precisely than in the text; although the points cannot be brought out without assuming doctrines which have to be explained afterwards.

1. It is alleged, in the text, that the propositions ½E and ½O leave open the universally prevailing alternative of the excluded middle: " Our given X's either are or are not Y's." The quantity of the subject being here indifferent, let the universal proposition ½E be taken for illustration: " The X's (any X's) are not some or other of the Y's." (1.) This is not inconsistent with the assertion (A) that " All the X's are some or other of the Y's:" for, though the X's are not some or other undetermined Y's, they may be some

other Y's also undetermined. I may say that "Men are not some or other of the objects we call imperfect beings," without contradicting the true assertion, that "Men are some or other of those beings." I may say that men are not to be found in some undetermined part of the class of imperfect beings, although I know that men are to be found in some other undetermined part of the class. (2.) The proposition is not inconsistent with the assertion (E) that "None of the X's are any of the Y's:" indeed, the assertion that "The X's are not some undetermined Y's," is implied in the assertion that "The X's are not any Y's:" it is a clumsy subalternate. If I choose to assert, "Men are not some or other of the objects we call stones," I assert a part of the wider truth, that "Men are not any of those objects." The assertion that men are not in some undetermined part of the class, is covered by the assertion that men are not in any part of the class. (3.) Accordingly, our proposition is consistent both with the proposition of inclusion, "All the X's are Y's," and with the proposition of exclusion, "The X's (any) are not Y's." It leaves untouched the disjunctive proposition, "The X's are either Y's or not Y's." This proposition collects the whole of our positive knowledge of the X's; and that knowledge is really no knowledge at all. Anything whatever must be either Y or not Y; so therefore, of course, must our X's be. (4.) We may regard the proposition as a fact of naming. The question then is this: Is our predicate a name which may be given to the things for which our subject is another name? May the things which, looking at them in respect of certain of their attributes, we call X's, be also called Y's, in respect of certain other attributes? The question cannot be answered. Our predicate Y may be a name, both for our X's and for other things; or it may not be a name for any of our X's.

2. It appears, then, that of the three kinds of propositions which have, in the text, been asserted to be the only ones available for the explication of implied knowledge, there are two towards which the propositions in question are indifferent. Such a proposition is consistent with a proposition of inclusion: it is consistent with a proposition of exclusion. Now these two are the only kinds of propositions taken account of in the received logical systems. There-

fore, if a proposition were given in either of the two new forms, we could not, with the same subject and predicate discharging the same functions, evolve, for the application of the common rules, either an affirmative (A or I), or a negative (E or O).

3. It must be allowed, however, that our propositions do give us hold of a predication of one sort. They are inconsistent with our third kind of propositions, those of constitution. If it is true that our X's are not some or other of the Y's, it cannot be true that they are all the Y's: since the X's are different things from some Y's or other, they cannot be identical with all the Y's. If, then, we assert, in our new forms, that "The X's (any or some) are not some Y's," we cannot, without self-contradiction, assert that "The X's (all or some) are all the Y's." Therefore, ½E and ½O severally contradict our A^2 and I^2. If we are to gain an expression for the contradictory thus implied, and if we are still to adhere to our given subject as subject, we evolve such an assertion as this: "The X's (all or some) either are not Y's, or, if they are Y's, they are not the only Y's." We are still forced into the disjunctive proposition, if we are to express all that our relation implies. But the positive member of our alternative has now received a negative limitation; and in this limitation lies the only force of our proposition as an element of knowledge.

4. It has such a force. For there is extricable, from our newly-gained disjunctive, a proposition in a received form, which, while it leaves open the A and I like our datum, does also like it contradict categorically the A^2 or I^2. It is expressible so as to cover both of the challenged forms; "There are Y's which are not our X's = Some Y's or other—are not—our X's." (1.) If our given proposition was "Some X's are not some Y's," our evolved proposition is, "Some Y's are not some X's." This assertion fulfils the conditions above alleged; but, being a re-emergence of the challengeable form, it may be passed over. (2.) If our given proposition was, "The X's (any X's) are not some Y's," the evolved proposition is, "Some Y's are not any X's." This proposition calls for particular examination. In the *first* place, it is not inconsistent with the assertion that "All X's are some Y's" (A). It is true

that "Some men are not (any) sages;" though it is also true that "All sages are (some) men." *Secondly*, it is not inconsistent with the assertion that "No X's are any Y's" (E): indeed, the assertion that "Some stones are not (any) men," might be worked out of the assertion that "No men are (any) stones." *Further*, it is plainly inconsistent with the assertion that "All the X's are all the Y's" (A^2): and thus it is also inconsistent with the I^2. If there are any Y's besides the X's, we cannot say, consistently, that the X's, or some of them, comprehend all the Y's. *Lastly*, the proposition we have thus gained is in one of the received forms: it is an O, a particular negative with distributed predicate.

5. Our extricated proposition, then, is a proposition of exclusion, a workable assertion of non-identity. But mark how it stands relatively to the point from which we started. Our terms have exchanged functions. Our subject has become predicate; our predicate has become subject. Our given subject, that term which was proposed for determination, was "the X's:" we sought to determine that term negatively through the term "some Y's." We failed in the attempt: we have failed even now. What we have been able to do is, not to determine X through Y, but to determine Y through X. We have asserted, in our new proposition, nothing about X as subject: we have asserted something about Y, shelving X into the office of predicate. In short, when we endeavoured to use the proposition as given, we discovered that we had grasped it by the wrong handle: when thus treated it slipt away from us. We have next seized it from the opposite side; and now our hold is firm.

6. Technically described, our change of position has been this: we have Converted the given proposition. Our O fulfils all the logical conditions of a valid converse. We were unable to extricate from our datum, either by affirmation or by negation, any determination as to our subject through our predicate. But conversion has yielded us a negative determination of our predicate through our subject.

7. If, then, it were conceivable that there should be actually given a proposition in the seventh form, our only feasible method of procedure would be founded on the theory, that our datum is a

product of perplexed and mistaken thought. In form denying the predicate of the subject, but not really amounting to such a denial, it does really imply an assertion in which the subject is denied of the predicate. Any one who should think in such a form, must, we would assume, have mistaken the substance for the attribute; and contrariwise. We should have to evolve the positive thought, and make it distinct, by transposing the terms. If such an expedient is not proposed in the text, it is because it does not seem to be the fact, that confusion of thought ever does show itself in this out-of-the-way guise.

8. The strongest claim of the seventh form to admission into the syllogistic system, rests on this relation between the proposition and its converse. But the claim takes the case from the side opposite to that on which we have hitherto looked at it.

It is a received and unchallengeable logical doctrine, that, all negatives being held to distribute the predicate, the O of the common scheme does not admit conversion into any proposition of that scheme: (its conversion by contraposition is really a conversion, not of the O, but of an I inferred from it). In a just conversion, while the quality of the proposition must remain unchanged, the terms must be transposed as wholes, quantity included. Given, then, "Some X's—are not—any Y's" (O); the subject "Some X's" cannot do duty as predicate. The impossibility of directly converting O, cripples seriously our dealing with two of the syllogistic moods, Baroco and Bocardo.

If our seventh form be admitted, it gives instant relief. It yields a converse of O: "Any Y's—are not—some X's." All the four kinds of propositions are now convertible; and the two formidable syllogisms are lowered from their bad eminence.

9. The question is, what has been gained by this transformation of the O? Why, we have displaced an assertion expressing a pregnant, though narrow thought, and have erected in its place the expression of an empty shadow of thinking. We had received a negative determination of our limited subject; we have transformed it into a total want of determination of our more extensive predicate. We have been allowed to start from a judgment which, though the

narrowest that is knowledge at all, is yet, within its small bounds, a knowledge precise and usable: we have wilfully thrown ourselves back into a position of pure doubt, a position from which we cannot rise unless by returning to the very point we had deserted.

10. The particular negative (O) of the received doctrine is the weakest of all possible judgments. The relation which it asserts is the narrowest that can yield any knowledge whatever: the amount of inference it allows is smaller than that given by any other proposition. One of the most telling proofs of its feebleness is the fact that, while it does deny something of the subject, it does not really either affirm or deny anything categorically of the predicate. The old logicians have recognised this fact, in pronouncing the O to be inconvertible: and, in the face of the temptation held out by a dazzling promise of increase in the forms of predication, the belief forces itself on us, that the old logicians were in the right.

11. One other query may be hazarded, bearing on that thorough extrication of the two wholes of the concept, the application of which to the theory of the syllogism is so admirable and original a feature in Sir W. Hamilton's system. Sufficient data being supplied, as they are in the premises of a syllogism, we ought to be able to determine, as to each of the three syllogistic propositions, in which of the two wholes it predicates. E and O are easily dealt with as propositions of inclusion, when the contradictory of the predicate is taken as the class. But how as to $\frac{1}{2}$E or $\frac{1}{2}$O, if these present themselves? Are they in any way thinkable, as predications either in extension or in comprehension?

Note II.

Hints for the Interpretation of Propositions.

I. When forms of expression, designed for the excitement of imagination or emotion, are to be logically used, they must either be translated into assertions expressive of pure thought; or, if any of the ideas denoted cannot be so translated, these must be neglected,

as not logically cognizable. Thus, all figurative phrases must be brought within our grasp by direct assertion of the relation they imply: "All flesh is grass," finds its equivalent in "Man is as fading as grass." Exclamations, again, are assertions intensified in meaning through indications of emotion. The emotive or intensive phrase may be made logically available when it is not a sign of quantity, but not when it is: "How miserable are some men!" is fairly interpretable into "Some men are very miserable;" "How many men are miserable!" cannot find a direct equivalent. Again, assertions made passionately, fall often, both in oratory and in common speech, into the form of question: the Interrogation is the favourite figure of Demosthenes. The assertion extricable is the answer; the quality of which is opposed to that of the question.

II. When we pass to assertions which may be taken to be already expressions of pure thinking, the first point that arises is the character of the *Copula*, raising the question of Modality. Pure categoricals are such as have been considered in the text. Modals have, as a copula, not the verb "to be" by itself, but this with some phrase which adds to or restricts its meaning. There seems to be no reason for questioning the sentence which excludes modals from logical treatment; but they are often interpretable into a shape which gives effect to the modal element, through its incorporation into one of the terms.

1. Treated in the most systematic way, modality is of three kinds, giving Kant's Judgments Problematical, Assertory, and Apodeictic (or Demonstrative). These are founded on the relations of possibility, reality, and necessity, and are expressible in the copula (affirmatively) by "may be," "is," and "must be." When considered psychologically and metaphysically, these varieties of thought are very important. In this view, "may be" is an expression, not of knowledge, but of a doubt which may or may not lead to knowledge. The imperative "must," on the other hand, seems to give voice to the only form, in which we can directly think the necessity attending our immediate cognition of *à priori* truths. That necessity cannot pass into the form of universality through "all," until we have both represented the primitive cognition,

and determined, rightly or wrongly, the sphere in which the law works. When this step has been taken, the "must" becomes quantitative; and the copula may be "is." Now, unless when the method next to be noticed is accessible, the hint just thrown out appears to indicate the only manner in which the threefold modality can be regarded as bearing on pure categorical predication. The "is" being accepted as the copula, the "must," when interpretable at all under this condition, signifies the universality of the subject; the "may be" its particularity. "Body must occupy space;" that is, "All bodies occupy space." "Body may be visible;" that is, "Some bodies are visible, some (as gases) are not."

2. Modality is frequently constituted by qualifying phrases, which (as is often true also of the "must," and "may") are easily transferable from copula to predicate. "John—is probably—dead," becomes pure as "John—is—a person probably dead;" and this transformation of the proposition would commonly, though not invariably, fit it for use in a given case.

3. The most stubborn kind of modality is made by the element of *time*, which often resists successfully all attempts at displacement. The logical copula merely connects subject and predicate, on the hypothesis that both denote objects which do or may exist. "X is Y" has logically no more meaning than this: "If X is, and if Y is, X is Y." Even our "is," because suggesting the idea of time, is not theoretically perfect as a symbol of the relation between the terms: there is a strong temptation (which must be resisted for avoidance of counterbalancing evils) to the substitution for it of mathematical signs like the =. But, when an assertion is made as to the past (and the same thing might be said of the future), we cannot, by any exertion, shake out the actuality which clings to the root of it. Allegations of historical facts cannot become pure categoricals, without destruction of their essential import. The truth is, that narrative propositions, *qua* allegations of past individual facts, are not adequate data for reasoning that embraces classes of objects and their laws. Nor are they really so used. The simplest general reflection that can insinuate itself into

the body of a history, will, if founded on an incident or characteristic appearing in the story, be found to have silently transformed the individual fact into an instance exemplifying some principle, holding for all time, and expressible, though not expressed, in pure categorical form. When, in historical writing, an inference is drawn from one individual fact to another, it might be logically tested in a fashion which, though of the roughest, and involving inquiries extra-logical, may sometimes be useful. It will, in particular, save needless trouble, when we encounter an argument in which the copula does not always appear in the same tense. Let us ask ourselves, after having examined the matter of the propositions, whether the change of time is essential or inessential to the mutual relations of the terms. If it is inessential, we may shut our eyes to the discrepancy. The test is stood successfully by such an argument as this: "Sages deserve fame (that is, always); Socrates was a sage: therefore Socrates deserved fame (or even Socrates deserves it)."

III. In a third class of propositions, the difficulty arises from the *Terms*. Each proposition of this class is resolvable into more propositions than one; though it is a question, to be answered only from scrutiny of the use the assertion is put to in a given case, whether it is to be so resolved, or to be treated as one assertion. Such propositions are describable by old names, as "*propositiones compositæ*," or compound; or as "*exponibiles*," in respect of their susceptibility of analysis; and some of them have, by certain logicians, been regarded as a species of modals.

All such forms are instances of the abbreviations to which language has recourse, in its vain endeavour to keep pace with the rapidity of thought. The varieties are as indefinitely numerous as the kinds of the occasions. The following are a few of those which occur most frequently in immediate relation to trains of reasoning :—

1. A Hypothetical proposition is the condensed expression of an inference, without categorical assertion of the premise: "If X is Y, Y is Z." The propositions called Inferential and Causal have the same import, with this difference, that they categorically assert the premise, sometimes through a participle: "X is Y, therefore

Y is Z: Y is Z because X is Y: X, being Y, is Z." If the inferential relation appears on the face of the compound proposition, it may be dealt with logically: if it does not, if it merely asserts the connection of two facts not formally related through a law of thought, the relation lies beyond the logical sphere.

2. A Disjunctive proposition, if affirmative, is equivalent to the assertion that one or another of two or more categorical propositions is true: "A is either X, or Y, or Z: either X, or Y, or Z, is A." If negative, it denies each of the alternatives: "A is neither X, nor Y, nor Z: neither X, nor Y, nor Z, is A."

Both hypotheticals and disjunctives will have to be treated more closely in a further stage of our progress.

3. The propositions oftenest called Copulatives are categorical affirmatives, in which either term, or both, are resolvable into simpler terms, co-ordinates of each other: "A is X, and Y, and Z: X, and Y, and Z, are A." If the terms are common terms, A may be the name of a class, X, Y, and Z the names of sub-classes constituting it; and in this case the proposition is treatable as an equivalent of the affirmative disjunctive. Such a proposition is an A^2. Very frequently this analysis is inapplicable: "Honesty and industry are virtues: industry is commendable and self-rewarding." But an assertion having the same form is often, in reality, one assertion only, the complex term being taken collectively: "Honesty and industry give promise of success;" that is, all combinations of honesty and industry give such promise.

4. Exclusive propositions are marked by such phrases as "only," connected with the predicate, and always (if we mistake not) truly referable to it: "All X's (or some X's) are the only Y's."

It has already been observed that, in making affirmative assertions of this sort, we really bring into actual use the questioned forms A^2 and I^2; and it has been pointed out, likewise, that another proposition may be held as implied. The full expression of the thought, in this view, gives these two predications: (1.) "All (or some) X's are some Y's:" (2.) "Things which are not Y's are not X's."

A proposition exclusive by negation would be such as this: "The

X's (or some X's) are not the only Y's." Such an assertion is not expressible by one proposition, in any of the eight forms of predication: it is not so even by ½E or ½O, unless the "some" of the predicate were to be interpreted definitely. Its whole signification is reachable only through analysis into its two factors: (1.) "All (or some) X's are some Y's:" (2.) "Some Y's are not any X's."

5. Exceptive propositions are marked, in the subject, by phrases like "but, except, unless, besides," which are equivalents of "not;" while "only," too, has certainly an exceptive force (of negation) when joined with the subject.

Accordingly, the affirmation "All objects besides the X's are Y's," is directly equivalent to the affirmation, "All objects which are not X's are some Y's (= All Not-X's are Y's)." But it presupposes or implies also the negation: "The X's are not any objects which are Not-Y's (= The X's are not Not-Y's)." So the negation, "No objects but the X's are Y's," is exponible into the expressed negation, "No objects not X's are any Y's," and the implied affirmation, "All the X's are some Y's."

When either exclusive propositions or exceptives appear in a chain of reasoning, it will almost always be found, that the expressed factor is that for the sake of which the allegation is introduced, and that no use is made of the implied one. But the implication requires to be remembered, in case of its emerging so as to cause a fallacy; and confused thinking may be made still more confused, through these or any other of the complex kinds of propositions.

6. A Comparative proposition presupposes another, with which it might stand undissected. The assertion that "Washington was a greater man than Napoleon," assumes that "Napoleon was a great man."

7. Restrictive propositions are not always analysable on the same principle. Sometimes such a proposition is, even in the shape in which it is given, a true and simple categorical, having a term which denotes (perhaps not so neatly as might be) a very complex idea. Often,—as in the case of the "reduplicativæ" of the schoolmen,—it is virtually an inference. "Man, so far as he is an ani-

mal, is mortal," seems fairly interpretable into "Man, being an animal, is mortal:" and this, again, is a causal proposition.

IV. Sometimes there insinuate themselves into reasoning, assertions of a kind, which it is not difficult to dispose of when their true character is understood. Their distinctive feature is not condensation, but expansion. They are best illustrated by propositions usually called Adversative, which assert, in any of several ways, a contrariety between one proposition and another.

1. The reason for adoption of the adversative form is frequently the desire to use that excitative power over imagination and feeling, which is possessed by the antithesis. In such a case, it will seldom be possible to determine the logical bearing of the proposition, until it has been thrown into another shape; and the choice may lie between any of several shapes, one of which only will exhibit the intended function of the proposition as a step of inference. Thus, if a proposition like this were to find its way into argument: "Life is short, but art is long," it would probably have to be interpreted as meaning, either that life is too short for the mastery of art; or that the complete study of art is too arduous for one short human life.

2. The adversative factor of the proposition may be merely explanatory or limitative. It may have been introduced in order to make it quite clear, either that certain cases are excluded from the scope of the principal assertion, or that certain cases are included in it. The context ought to show plainly, which of the two factors is the assertion founded on in the reasoning, and which is a mere gloss not entering into the argument at all.

CHAPTER II.

The Laws of Categorical Predication through Common Terms.

44. Let two terms be given, with the postulated explanations; and, the question of inference being postponed, let it be required only to predicate or form propositions with them. Logic can work the problem no further, than by exhibiting all the propositional forms in which it is possible to combine the terms, whether affirmatively or negatively. Which of the propositions, if any, would be true in respect of the relation between the objects signified by the terms, or which false in respect of that relation, is a point to be determined, as to each proposition considered by itself, through knowledge of the matter, and not otherwise.

Mixed predication, through terms singular and common.

When any of the terms are singulars, the propositional scheme which has been examined must, as framed with exclusive reference to common terms, be in part inapplicable. But, the singular term being treated as a common term distributed, forms will arise which are virtually equivalent to certain of our six. Some of the six are without any such parallels.

In the first place, both terms may be singular. In this very simple and unfruitful case, the only possible predications are equivalents of A^2 and E.

A greater variety of forms, as well as a wider possibility of inference, is produced by Mixed Predication, in which the given terms are, the one a common term, the other a

singular. Now, we do not naturally think a singular term as predicate, either affirmatively or negatively, when the other term is common. Accordingly, with reference to the functions of the terms, a distinction may advantageously be taken, between forms which do spontaneously present themselves, and others which are gained only through logical analysis, or through a process of reflection virtually amounting to it.

1. Two of the six forms are in all cases excluded: I, by the impossibility of particularizing the singular term; A^2 by the impossibility of thinking an individual as constituting a class.

2. Equivalents of the other four forms are admissible, but under dissimilar conditions. (1.) The E is possible with either position of the terms. It occurs naturally and continually with the singular as subject: "John is not an (any) archbishop." With that term as predicate it occurs, perhaps, never, unless as the result of a scrutiny for purposes really logical; as, for instance, when we wish to change the form of a given argument. Technically speaking, it arises through conversion. (2.) The occurrence of A is both possible and incessant, the singular being the subject: "John is a (some) good man." (3.) The I^2, possible only when the singular is the predicate, is in the same predicament with the second variety of the E. It is, although the received logical rules disguise the fact, the just converse of A. (4.) The O is possible with the singular as predicate; but this, the weakest of all predicative forms, is stricken with more stubborn barrenness through the inflexibility of the singular. Probably the proposition is without example in ordinary and unanalytic thought; and its uses of any sort must be very rare.

These mixed forms are evidently ruled, with exceptions

neither many nor obscure, by the same laws which govern predication through terms all of which are common. Inference from them, also, both immediate and mediate, is similarly placed towards inference proceeding purely through common terms. It seems sufficient, therefore, to have indicated, as here, the forms of mixed predication. No attempt will be made to assign for them any special laws of inference.[1]

Predication, through common terms, in extension and in comprehension.

45. Our attention will henceforth be directed exclusively to Predication and Inference through Common Terms.

Predication through common terms is limited, in more

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[1] Here arises a psychological question. It was noted, in the introduction, that, in the German nomenclature, the word "thought" does not include any cognition that is not discursive. It might have been added that, by Kant, the name is specificated a step further, so as to signify only "cognition through concepts." If the word is to be thus narrowly understood, it can scarcely cover predication or inference in which any of the terms are singulars; yet these are logically treatable, and must therefore be admitted to signify thoughts. Some of the Kantist logicians have sought to remove the difficulty, by maintaining, that the significate of a singular term becomes a true concept, whenever it is an element in a judgment logically analysable. But, surely, such a theory ignores the distinctive character of the concept. The singular term is the symbol of an image (*Bild*), representative of an intuition (*Anschauung*) real or possible. That the representation is partial, incomplete, is nothing more than what seems to be true of every image. That the representation, if denoted by words, is symbolic, is a fact which cannot change its objective reference. Conception is necessarily symbolic; but symbolic cognition is not necessarily conception. Some of Mr Mansel's speculations bear closely and instructively on this question.

quarters than one, by a corollary already noticed as following from the primary law of the concept, that is, the Inverse Ratio of the terms.

We cannot, in one and the same judgment, analyse a concept, or make a predication giving the result of the analysis, in both of the wholes which together make up the synthetic totality of the concept. "We must either think explicitly in extension, and imply comprehension; or think explicitly in comprehension, and imply extension."

Every term, with which there are given to us materials permitting predications of it in both wholes, must be thought as standing in an ordinated series, of which it is not either extreme. Upwards from it in extension there must stand terms, one, or more than one, which are names of classes that contain, step by step, more objects, because the objects possess, step by step, fewer attributes. Downwards from it in extension there must stand terms, one, or more than one, which are names of classes that contain, step by step, fewer objects, because the objects possess, step by step, more attributes. The term from which we start takes, naturally, as antecedent, the function of subject in predication. We may find predicates for it by looking either upwards or downwards. But we cannot look both ways at once: and we gain one predicate or set of predicates by searching in the one direction, another predicate or set of predicates by searching in the other.

The result, then, is this. Every proposition, framed with two common terms, must be either *a predication in extension or a predication in comprehension.* It must be, either, a predication of the subject in (or out of) the extension of another term, which is the predicate; or a predication of the subject in (or out of) the comprehension of another

term as predicate. It cannot be both. We predicate of a term, as subject, in the extension of the predicate, by affirming of it a term denoting a more extensive class. We predicate of a term, as subject, in the comprehension of the predicate, by affirming of it a term denoting a less extensive class. Thus, of the subject-term "animals," we predicate in extension by affirming of it "organized beings," as predicate: we predicate of it in comprehension by affirming of it "birds."

Suppose a proposition is given, but only one. If, as in those examples, we happen to know the actual relations of the objects denoted by the terms, we can say, peremptorily, in which of the two wholes the predication is. But the question cannot be determined in the absence of such information. No assistance is yielded by any forms of expression, either usually occurring or at all likely to occur. Nor would it be easy, if so much as possible, to devise technical expressions adequate to the purpose. Abstract phrases, into which predications in comprehension are analytically resolvable, are in very many instances not extant: and it seems impossible so to mould them, that they shall fully denote the quantity of the terms. We think and speak, by preference, concretely; and we thus suggest predication in extension. If an assertion has really the opposite character, the fact must be inferred from data which are wanting in the case supposed. We say, in extension, "All animals are organized beings." But we say, likewise, in comprehension, "Some animals are birds."[1]

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[1] The only available forms of expression (and even these but partially sufficient), would be gained through explicit signature of the quantity of predicates.—In dealing with the references which

Predication with two common terms given; and with terms given in an ordinated series.

46. Suppose, then, that there are given two common terms, under the conditions postulated, but without any further datum. Logic, if called on only to form one proposition, can determine nothing more than this: that the only alternatives of predication are yielded by A, I, E, and O; or by A^2 and I^2, also, if these are admitted.

Much closer determinations, indeed, can be reached, if any one of those propositions be supposed to be formed, and assumed to be either true or false. All the forms are so related to each other, through direct applicability of the logical axioms, that the assumed truth or falsehood of any one of them warrants us, with certain restrictions, in asserting the falsehood or truth of each of the others. These mutual relations are, by many logicians, considered, under the name of Opposition, as affections of the proposition; and the laws governing them are treated as laws of predication.

Strictly taken, this evolution of one proposition from another is inference, not mere predication: and other kinds of inference from one proposition have also to be examined. All will be taken together, when, at our next and last step, we pass beyond the study of the proposition.

But the difference between predication and inference is

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concrete and abstract thinking severally have to the two wholes of the concept, Mr Karslake has broken up ground which may hereafter prove to be very fertile. Again, the combination of concrete terms and abstract in the same proposition, the one as subject, the other as predicate, is, if it occurs spontaneously, a symptom of confused thought. If it is introduced wilfully, as it sometimes is in the logical treatment of given examples, it generates the same confusion, of which it is a natural expression. (See Hamilton's *Discussions*, p. 646.) "Whiteness," says Occam, "is not white."

nothing beyond a difference in the form of the data. We predicate in framing a proposition from given terms: we infer in framing a proposition from one or more given propositions. It is a doctrine to be insisted on, that inference is merely predication taking place in more steps than one; and that all the laws of inference are but variations, designed to meet greater or less complication of materials, of the logical axioms, which are strictly laws of predication.[1]

This is one reason for considering exactly, from the position now reached, the laws which govern the formation of propositions from given terms, in a class of cases differing considerably from that which has just been laid aside. Let there be given common terms, either two, or more than two; and let there be given with them, not the quantitative signs, but, which is much more, an explicit *ordination of the terms.* The ordination may be indifferently in extension or in comprehension, provided only we be informed in which of the two it is.

Is such an ordination ever actually given? And is it recognisable without interpretation of the terms? Both questions must be answered in the affirmative. On the one hand, it is, as we shall immediately see, the datum of every definition and of every logical division. On the other hand stands a fact which concerns the logician still more nearly. In every syllogism, having premises which allow any inference, there are given three terms: and, if

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[1] See, afterwards, *Doctrine of Inference,* chapter i.—Compare Mansel, *Prolegomena,* pp. 196, 207. By Twesten (*Die Logik, insbesondere die Analytik,* 1825), all the forms of thought are exhibited in an ascending series, whose members increase in complexity according to the character and number of the data.

these terms are common, there is implied, and easily extricable, an ordination of these, the discovery of which does not require any scrutiny of their meaning. The ordination being gained, we may, by combining the terms two and two, form, not only the conclusion of the given syllogism, but also several or many other propositions. All these results are accessible through canons, which are nothing but corollaries, the simplest and most obvious, from the principle of the concept, the law of the inverse ratio.

We shall have laid the broadest and firmest foundation for a just understanding of the character of syllogistic reasoning, if we satisfy ourselves, at present, that all syllogistic conclusions are attainable through direct comparison of the terms of the argument, without the explicit statement of the relations of the terms in the form of propositions. All Inference, whether Immediate, that is, from one proposition, or Mediate, that is, from propositions more than one, is merely an explicit assertion of the implied relations of terms. The process is called inference, when the relations of two or more terms are given as already explicated in propositions; and when the problem proposed is the analysing of those propositions, for the purpose of discovering what other relations are implied in the assumed ones, and may, therefore, be expressly educed from them. The process would not be called inference, but predication, if the relations of the terms were given as unexplicated; which is the case when the terms are only described for us as holding certain places in an ordinated scale. The theory of reasoning is not reduced to its utmost simplicity until it has been made evident, that the process, into whichever of the two forms the data may throw it, is really one and the same.

Therefore it is desirable that we should, at once, put our-

selves in possession of the laws which regulate predication through common terms, both in extension and in comprehension. Another reason is this. When we come to study inference specially, its two kinds, immediate and mediate, must be taken separately. Some of these laws of predication bear on the one kind of inference, some on the other; consequently, if not now collected, they would appear only as isolated theorems. Some of them, too, would not come clearly into light at all.

I. *Predication in Extension.*

47. Let there be given, as ordinated in extension, from highest to lowest, a series of two or more terms: and let it be required to predicate with these in extension, both affirmatively and negatively; that is, let it be required to assert of one term, given as subject, that it is in, or out of, the extension of another term found as predicate. The possibilities of affirmation and negation, and the quantitative determinations of the terms, are set forth in the following rules.

The laws c predicatio in extension: affir mation an negation.

(I.) AFFIRMATION.

I. Of any subordinate term, there may be affirmed any term positively superordinate to it, either immediately or mediately. This is the one universal canon.

Co-ordinates are here excluded from consideration. All the objects of the subordinate class are, through the relations involved in the character of concepts, included in each of the classes positively superordinate to it. All the objects which, in respect of a certain attribute, are called by the name of the subordinate class, are some or other of

the objects which, in respect of other attributes, receive the names of the superordinate classes. If the given series, ordinated from highest to lowest, be X, Y, Z, we may affirm that "All the Z's are some Y's;" that "All the Y's are some X's;" and that "All the Z's are some X's."

II. Of a subordinate term, given as distributed, a positive superordinate may be thus affirmed, through any of several presuppositions; and these, if successively engrafted on each other, will throw the process into several different forms.

(1.) Of a subordinate term there may be affirmed, universally, any term thought as superordinate to it, immediately or in the first degree. There is thus formed a simple predication of identity in A; as, "All the Z's are some Y's"; or, "All the Y's are some X's."

(2.) In such a proposition there is implied another. The subordinate being given as distributed, there may be affirmed of it, as undistributed, the same superordinate. If all the objects of the lower class are included in the higher class, some at least of them must be so. We have thus from one predication of identity derived a second, from an A an I: "Some of the Z's are some Y's;" or, "Some of the Y's are some X's." The process is an immediate inference by subalternation.

(3.) Of a subordinate term (distributed or undistributed), there may be affirmed, in A or I, a term thought as superordinate to it mediately in the second degree; that is, a term thought as immediately superordinate to the immediate superordinate of the given term. The objects (all or some) of the given class Z are identical with some or other of the objects of the class immediately superordinate, Y; and all the objects of the class Y are identical with some or other of the objects of its immediate superordinate, Z: there-

fore, necessarily, there are, in the intermediate class Y, objects which, in respect of one attribute, receive the subordinate name Z; while, in respect of another attribute, they receive the superordinate name X. If all the identities which are tnus discoverable are explicitly enunciated, they yield the three following propositions:—"All (or some) Z's are some Y's (A or I)"; "All the Y's are some X's (A)"; "All (or some) Z's are some X's (A or I)." The third assertion of identity is elicited from the first and second, considered in relation to each other. "The Z's (all or some) are identical with some or other of the Y's (A or I); and all the Y's are identical with some or other of the X's (A or I): therefore the Z's (all or some) are identical with some or other of the X's (A or I)." In this explicated form of all the steps, the process is a mediate inference of the least complex kind. The three propositions constitute an Affirmative Syllogism.[1]

(4.) Of a subordinate term (distributed or undistributed), there may be affirmed, in A or I, a term thought as superordinate to it in any degree beyond the second. When such a process is evolved at every step, it is found to consist in repeated predications of identity: it is, in fact, an extension, through higher degrees, of the process of syllogistic

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[1] Here, accordingly, we hover very near to debateable ground, which must afterwards be fairly traversed. If the terms of a conclusion are thought as ordinated in one degree, it is reached through simple subalternation. But the question may be raised, even now: whether, supposing it is only through inclusion in Y that we do actually think the inclusion of Z in X, both of the steps constituting the premises must necessarily be thought explicitly in the form of judgments; or whether one of them may not, without detriment to the process, continue unexplicated and only implied.

inference. The series of propositions is called by logicians a Sorites; the ultimate conclusion of which must, on such data as these, be affirmative, but may be either universal or particular.

(II.) NEGATION.

As affirmation in extension rests on the law of identity, applied to objects thought as included in classes, so negation in extension rests on the law of difference, applied to objects thought as excluded from classes. If, of either Z or Y, I am entitled to deny X, this must be because I think of X as being something different from Z or Y: X must be thought as equivalent to Not-Z or Not-Y. It follows, that negation is not applicable to a series of terms positively ordinated, unless by substituting, in the predicate, the contradictory of a term for the term itself; as if, for "All X's are some Y's," we should take the equivalent negation, "The X's are not any Not-Y's."

But negation finds a place without this expedient, as soon as there is incorporated into our positive series of terms a Co-ordinate of any one of them. Terms co-ordinate are thought as being, not indeed absolutely, but within the given sphere of thought, contradictories of each other.[1] Thus, if our thinking is limited by its hypothesis to the class of

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[1] This inevitable limitation of the sphere, within which the laws of difference and excluded middle must work when the terms constitute an ordinated series, is strongly put by Trendelenburg, and grounds his attack on division by dichotomy. (See his *Logische Untersuchungen*, vol. ii., p. 317, and elsewhere; and his *Elementa Logices Aristoteleæ*, § 58.) It is also very firmly apprehended by Mr De Morgan. (*Formal Logic*, p. 38, and *passim.*)

objects which we call "organized beings," that class may be further thought as containing only two sub-classes, "animals" and "vegetables:" hence all organized beings which are animals, are thinkable also as being "not-vegetables;" and so the opposite way. Co-ordinate classes must admit of being so thought, if they are to obey the law which makes such classes to be exclusive of each other. Any two classes being thought as co-ordinate, all the objects of each are thought as having some attribute wanting in all the objects of the other. Therefore any object which is in the one class cannot be in the other: the two terms must be names for two groups of objects totally different.

One special remark is required. If we either know the meanings of terms, or have received an ordinated series, we can determine peremptorily, as to any proposition framed with a higher term and a lower, in which of the two wholes of the predicate the predication is made. But every proposition denying one co-ordinate of another, may be regarded as being either in extension or in comprehension: for each of the terms excludes the other in both relations.

The rules cannot conveniently be grouped, like those for affirmation, under one canon covering all possible cases. But, throughout all of them, the co-ordinate takes the place which, in the affirmative rules, was held by one of the superordinates. Let X, Y, Z, be given in ordination as before, and let *a*, *b*, *c*, be co-ordinates of those three terms severally.

I. Of any common term, there may be denied universally any term thought as co-ordinate to it.

There is thus formed, on principles already explained, a simple predication of non-identity in E: as, "Any X's are not any *a*'s:" and so of the other terms.

II. Of any common term, there may be denied particularly any term thought as co-ordinate to it.

The process is an immediate inference by subalternation, yielding an O: as, "Some X's are not any *a*'s." Its principle is that of the corresponding affirmation.

III. Of a subordinate term (distributed or undistributed), there may be denied any of the co-ordinates of any of its superordinates.

There arise, in this way, when all steps are explicated, processes of mediate inference, corresponding to those for affirmation, only with substitution of a co-ordinate for the superordinate of a superordinate. The principle is very plain. The subordinate is included in the superordinate; from the superordinate its co-ordinate is excluded: therefore the co-ordinate is excluded from the subordinate.

(1.) Of the subordinate (distributed or undistributed), there may be denied, in E or O, any co-ordinate of its superordinate in the first degree.

A co-ordinate of Y will be signified by *b*, which is thus equivalent to "Not-Y." Our terms will then yield these three predications, the first of identity, the other two of non-identity: "The Z's (all or some) are some Y's (A or I); any Y's are not any *b's* (E): therefore the Z's (any or some) are not any *b's*" (E or O). The series of identities and differences is self-evident. The three propositions constitute a Negative Syllogism. Accordingly this kind of syllogistic inference is, when analyzed in reference to the wholes of the terms, resolvable into an ordination having a different character from that which produced affirmative conclusions. The terms rise by one step only; and the higher term of the two, instead of rising by inclusion into a

third, diverges by exclusion into the parallel or co-ordinate.[1]

(2.) Of a subordinate term (distributed or undistributed), there may be denied, in E or O, any co-ordinate of any term superordinate to it in any degree beyond the first. Such a process yields a Sorites, whose conclusion may be universal or particular, but must be negative.

II. *Predication in Comprehension.*

The laws of predication in comprehension: affirmation and negation.

48. We must, and do, predicate in comprehension as well as in extension.

Not only, however, is it true, as has already been remarked, that we naturally express ourselves in those concrete forms which are appropriate to extension; but, further,

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[1] It does not seem possible to escape from this result of the analysis, if the negative forms are to be preserved. Nor can it be said to trench, in the slightest degree, on that more exact analysis of the Syllogism, which will, by and by, be attempted on the same principle.

But, if it is insisted on that the syllogism shall exhibit the X, Y, Z, in a regular scale of positive ordination, the negative syllogism may be made to do so by being transformed into an affirmative one. This is effected through a process which we shall immediately become acquainted with,—Contraposition. The excluding of the subject from the sphere of the predicate, is equivalent to the including of it in the sphere (indefinitely wider) of the contradictory of the predicate. The proposition "The Y's are not any *b's*," thus becomes "All the Y's are some *Not-b's*." Our negative syllogism might, through this change, become affirmative thus: "The Z's (all or some) are some Y's (A or I); all the Y's are some *Not-b's* (A): therefore the Z's (all or some) are some *Not-b's* (A or I)." Our ordinated terms are now these: "*Not-b*, Y, Z;" and the analysis of the affirmative syllogism is exactly applicable.

we never do, naturally or spontaneously, either think or speak in systematic pursuance of that course of thought which predication in comprehension would signify. We think from objects as data; and we scrutinize their attributes only as enabling us to place the objects in classes, to think of them as amenable to laws. Predication in comprehension does not emerge spontaneously in reasoning, unless in conjunction with predication in the opposite relation, and with a view to the ultimate establishment of that other.

So far in the background does predication in comprehension lie, that it is only modern logicians that have given systematic attention to its bearings on any doctrines of the science; while even of these there is only one who has brought to light its highest results. The four received forms of propositions, on which exclusively the received logical system rests, do not allow correct expression for this relation of the concept: and inferences bearing on it, whether mediate or immediate, require one additional form before they can be enunciated so that their validity shall be self-evident.

With anticipation, therefore, of uses to be found hereafter, it is well that the laws of predication in comprehension should be briefly set forth. They do not require to be elaborated so formally as those of extension, with reference to which, mainly, the laws of inference will be expressed. But the right apprehension of them demands patient attention, on account both of the smallness of the assistance which the orthodox systems give towards it, and because of the difficulty we all have in seizing this relation distinctly.

If we adopt, for exemplification, the same three symbolic terms as before, the ordination of these must, by reason of

the inverse ratio of the wholes, take place in the opposite order. Ordinated in comprehension, from highest to lowest, they will stand thus: Z, Y, X. It will be convenient, also, to illustrate the sequence by predication with significant terms: as these, for the three in their order:—"Man, animal, organized being."

(I.) AFFIRMATION.

I. Of any term subordinate in comprehension, it is true, first, that there may be affirmed of it any term superordinate to it in the same relation; secondly, that the affirmation must be particular; thirdly, that the affirmation must, if its interpretation is to be exhaustive, be held to have its predicate distributed, that is, to be in I^2 not in I. The proof is easy.

Let our affirmation be this: "Some Y's are Z's: some animals are men." When we thus, in respect of comprehension, ascend in passing from subject to predicate, we do, by the same step, descend in extension.

(1.) From whichsoever of the two sides we regard the terms, it is clear that affirmation is possible. The attribute, whose possession by certain objects is intimated by the subject, is possessed also by all the objects named in the predicate. Terms rise in comprehension, and fall in extension, not by signifying fewer and fewer attributes, but by adding, at each step, a new attribute to the first. There cannot but be objects nameable by both terms.

(2.) The increase in signified attributes carries with it a decrease in contained objects. The predicate, as implying one attribute more than the subject, cannot completely fill the extension of the subject: the subject-class must contain, besides the objects that are in the predicate-class,

those objects also which are not in it as not possessing its attribute. The affirmation must be particular.

(3.) The distribution of the predicate becomes most promptly visible if we first affirm with the same terms in extension. We thus gain the assertion: "All Z's are some Y's: All men are some animals." The counter-relation is incompletely rendered, if it is held to yield anything less than an exact and complete reversal of this affirmation. The affirmation in comprehension must be an I^2: "Some Y's are all Z's; some animals are all men." The nature of the ordinative relation elicits clearly the same signification. It is true, of some of the Y's, not that they are a part of the class Z, but that they constitute the whole of it: there are, by the hypothesis, no Z's besides those that are Y's. There are certain objects which are "animals:" but these we can call only "some animals;" because there are other animals besides them. Of those objects it is not true, that they are the same objects with "some" of those we call "men," and different objects from "some other" men: it is true that they are the same objects to "all" of which we give the name "men." The interpretation of the affirmation as I, "Some animals are some men," is doubtless safe, as asserting within the truth. It might, also, be formally justifiable, if we were to read the quantitative sign as "some at most." But, first, this is not the logical reading; and, next, if it were adopted, the affirmation would violate the sound precept of the logicians —that every proposition shall explicate completely the relations implied in its data.

II. The affirmation being already particular, subalternate inference from it is not possible according to the received scheme. But, if we are to adopt the I^2, we must hold it

as admitting a subalternate, through a formal limitation of the predicate, implying a real limitation of the subject also. This subalternate is just the I, which usually takes the place of the I^2. "Some animals are all men:" therefore, also, "Some animals (but a narrower 'some' than the first) are some men."

III. Mediate inference is possible through affirmation in comprehension, as widely as through affirmation in extension. But it is expressible only through the admission of I^2, if the propositions are to contain on the face of them evidence of the validity of the process.

It is sufficient, for the present, to set down, in the relation of comprehension, the universal mode of the same syllogism which already exemplified the relation of extension, together with a parallel in significant terms. "Some X's are all Y's (I^2); some Y's are all Z's (I^2): therefore, some X's are all Z's (I^2)." "Some organized creatures are all animals; some animals are all men: therefore some organized creatures are all men." Breaking loose from almost every formal rule of the syllogism, this argument does not violate any one of its philosophical laws.

(II.) NEGATION.

I. II. Co-ordinates, when considered without reference to other terms in a series, are indifferent to the two wholes of the concept. It follows, that the same two rules which stand first and second for predication in extension, may hold, and for the same reasons, a corresponding place here.

III. Of a term subordinate in comprehension there may be denied any of the co-ordinates of any of its superordinates. But the denial must be particular.

In respect of the quality of the proposition, this rule is

proveable by the same considerations which established the parallel rule in extension. The limitation of quantity requires no illustration beyond those already given in this section.

In negation, as in affirmation, the mediate inferences thus formed might be, though they never have been, carried upwards from the simple syllogism into the sorites.

III. *The Transference of Predication from Whole to Whole.*

The laws regulating the transference of predication from whole to whole.

49. For the perfecting of our insight into the character of predication in extension and comprehension, it is necessary to consider cursorily a relation which must afterwards be scrutinized more minutely, as yielding one of the kinds of immediate inference.

It is self-evident, that we may not only predicate in either whole, but also transfer a given predication from the one to the other. It seems to be almost equally plain, that the process which is called Conversion is nothing else than such a transference. Its theory is not made complete until it is contemplated in that aspect. The rules of the process will immediately be assigned; but the foundation for them ought to be here laid, in a few theorems, which appear to require little, if anything, either of proof or of illustration.

(1.) Any two common terms may be ordinated in either whole; and ordination in either implies and yields ordination in the other.

(2.) Consequently, any two ordinated terms may yield either a predication in extension, or a predication in comprehension.

(3.) By reason of the inverse ratio of the two wholes, the terms must, in the two propositions, discharge opposite

functions: that which is subject in the one must be predicate in the other. If X is in the extension of Y, Y must be in the comprehension of X.

(4.) Consequently, again, if there be given a proposition which predicates in the one whole, it may, by a simple reversal of the functions of the terms, be transformed into a proposition predicating in the other whole.

(5.) The process of conversion is nothing else than such a transference of predication from a given whole into the other. The special rules of conversion find their principle in the law of the concept: they are merely adapted forms of those corollaries of that law, which regulate predication in the two wholes.[1]

[1] This view of the character of Conversion does seem, not only to flow, by consequence obvious as well as necessary, from the principle of the concept, but to be necessary for thoroughly grounding the theory of the process. But certainly, so far as we know, it has not been stated by any, even of those recent logicians by whom, in this country and in Germany, the mutual relations of extension and comprehension have, in their bearing on other logical doctrines, been most deeply probed.

CHAPTER III.

The Laws of Definition and Division.

The form and character of definition and division.

50. Affirmative propositions, having both terms distributed, have uses which give them a special scientific and philosophical value: A^2 is the form necessarily assumed by Definitions and Divisions correctly constructed. The character, likewise, of definition and division, is dependent on the doctrine of the concept. A definition is nothing else than a development of the comprehension of a common term, through terms lying above it in extension. A division is a development of the extension of a common term, through terms lying above it in comprehension. Both may be said to have for their purpose the making concepts more distinct; the one by evolving concepts in whose extension the given concept lies, the other by evolving concepts in whose comprehension it lies. It is a consequence flowing necessarily from the mutual and inverse relation of the two wholes of the concept, that its comprehension shall be made more distinct through its extension, and its extension through its comprehension. We determine what are the attributes of given objects, by finding what classes they may be thought in: we determine what objects are contained in given classes, by finding what attributes they may be thought as possessing.[1]

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[1] Division and definition have long been thus analyzed by the German logicians; the latter as an evolution of the comprehen-

The three stages in the development of ideas.

51. Our thinking of objects may pass through very many stages, on its way towards becoming a knowledge of the objects. The principal of those stages may be said to be three; and to these have been assigned names, the technical meanings of which, being specifications of the ordinary meanings, require some explanation. Our ideas of objects may be either Obscure, Clear, or Distinct.[1] *First*, Our idea of an object is obscure, when we are not able positively to distinguish it from other objects; when we are unable to determine the question of identity or non-identity. Such a thought of the object is not knowledge of it in any sense. *Secondly*, Our idea of an object is clear, when we are able to distinguish it from other objects; when we are able to determine the question of identity or non-identity. Our thinking of individual objects must rise to this point before we can be said to know them: and, while objects are contemplated merely as individuals, this point cannot be transcended. But, in whatever light objects are regarded, clearness in our thinking of them must have place if any further step is to be taken. *Thirdly*, Our idea of objects is distinct,

sion of a concept, the former as an evolution of its extension. Their theory of division is almost complete: their theory of definition is not so near to being so. It does not seem correct to say, as it is said by some (not all) of them, that the concept is made more "clear" by division, more "distinct" by definition: in the appropriated meaning of those terms, as explained immediately, increase of "distinctness" appears to be what is gained in both ways. (But see Mansel, *Prolegomena Logica*, pp. 186-194.)

[1] The distinction, currently applied in the German schools, and lately beginning to be familiar among us, is Leibnitz's. It is laid down in his *Meditationes de Cognitione, Veritate, et Ideis*, and illustrated in his *Nouveaux Essais*, book ii., chap. 22; (*Opera*, ed. Erdmann, pp. 79-81, 288-292).

when, besides being able to distinguish them from others, we are able also to distinguish the relations between them and other objects. The distinguishing of the relations between objects is attainable only through the detection and discrimination of their attributes, and the consequent distribution of them in thought into classes. Perfect distinctness of thinking, in this appropriated signification of the phrase, is evidently not attainable in regard to any object of human knowledge: and, as far as there are relations of an object which we cannot distinguish from others, our idea is indistinct. Distinctness, therefore, is relative,—relative to the purpose of our thinking: and the practical question in a given case is, whether, with reference to the purpose, the distinctness is adequate or inadequate.

Accordingly, obscure thinking cannot yield terms of any kind that shall be useable with intelligence. Clear thinking may be represented either by singular or by common terms. Thinking which, besides being clear, is also distinct, can be signified by common terms only.

Definition and division as making concepts distinct.

52. Having gained a clear idea of a class of objects denoted by a common term, we next seek to make that idea distinct, by evolving such relations to other objects as are implied in the notion of the class. Using our common term as subject, we attain a step in distinctness by each other common term which we are able to affirm as predicate of it. Such affirmation we can justify to ourselves through, but only through, a preconceived ordination, in which our common term is one of the members; and we have the affirmation when we place our common term either in the extension or in the comprehension of another common term. Further, when we place a term Y in either whole of another

term, as either X or Z, we do so really for the purpose of evolving an element in the other whole of Y. The interlacement and inversion of the two wholes are inextricable and constant.

On the one hand, looking upwards in the scale of extension, we place our given class in a higher class, which, besides our given objects, contains also others having certain of the attributes of ours. Thus we affirm, in extension, that "All Y's are X's:" that "All animals are beings organized;" that all animals are contained in the class of organized beings. In so predicating, we make our idea of "animals" more distinct, by evolving the fact that animals possess the attribute of organization; that is, that "organization" is a part of the comprehension of "animal."

On the other hand, looking downwards in the scale of extension, we place our given class in a lower class, which contains fewer than all the objects of our class, because all the objects it does contain possess attributes not possessed by all our objects. Thus we affirm, in comprehension, that "Some Y's are Z's:" that "Some animals are men;" that some animals possess the attribute humanity. In so predicating, we make our idea of "animals" more distinct, by evolving the fact that some animals belong to the class man; that is, that "man" is a part of the extension of "animal."

Thus we have made our idea of a given term more distinct by two steps in opposite directions, through our possession of two other terms, the one higher than it in extension, the other lower. By placing Y in the extension of X, we enable ourselves to infer that X is in the comprehension of Y. By placing Y in the comprehension of Z, we enable ourselves to infer that Z is in the extension of Y.

Hypothetical growth of a definition and a division: the first step.

53. Suppose our whole knowledge of a common term Y, or of the objects denotable by it, to be present to the mind in the implicative shape of a series of terms, ordinated in extension; suppose that the series stretches from our common term both ways, upwards and downwards; and suppose, also, that it embraces no co-ordinate terms. The knowledge thus implied would be completely explicated by two successive affirmations. In each of these Y would be the subject: while the predicates would be the other terms of the series; the higher terms in the one affirmation, the lower in the other.

Let our series be this: "Organized beings—Animals—Men—Europeans—Scotsmen;" and let it be understood as an implicit expression of the complex idea signified by the term "*Men*."

In extension we may affirm, that "All men are animals and beings organized." Analytically taken, the assertion is this: "All men—are—some of those beings who are both animals and beings organized; or, "All men—are—some of those beings who possess the attributes animal life and organization." We have evolved two attributes which are in the comprehension of the term "man." Conversion makes the assertion a predication in comprehension: "Some of those beings who possess the attributes animal life and organization—are—all men." Our proposition is, in fact, a definition. It is, doubtless, an unsatisfactory and imperfect definition; and it betrays its faultiness by the non-distribution of one of its terms. But it is the only definition of "man" which our data allow us to form.

In comprehension, again, we may affirm, that "Some men are Europeans and Scotsmen." Analyzing the assertion, we have it thus: "Some men—are—all those beings

who are both Europeans and Scotsmen ;" or, " Some men —are—all those beings who, while they are in the class Europeans, constitute the class Scotsmen." We have evolved two classes, both of which are in the extension of the term " man." Conversion makes the assertion a predication in extension : " All those beings who are both Europeans and Scotsmen—are—some men." Our proposition is, in fact, a logical division. It is an imperfect division ; and the non-distribution of one of the terms brings the imperfection to the surface. But it is the only division of " man" that can be developed from the data.

The definition and division we have formed are both of them imperfect : they want something they should have. But they may be said to be also redundant : they have something which, in most cases, they need not have. It is well to clear away the redundancy before scrutinizing the grounds of the incompleteness.

Definition and division at their second step of growth.

54. In attempting to frame either a definition or a division, we pay especial attention to two points of limitation. We aim at simplifying both thought and expression. Entertaining this design, we directly explicate those elements only of the idea, those relations only of the objects, which we foresee to be available in the subsequent progress of our reasoning. We leave undeveloped all elements or relations, which do not seem to have a prospective bearing ; and we do so with safety, if the elements we neglect cannot emerge as we proceed in thought.

Perhaps we are well acquainted with the objects compared ; while, also, our field of reasoning is not to spread beyond a few of their relations. In such a case we shall usually, even if we have antecedently thought out a long

series of ordinated terms, neglect all except one of the higher terms, or all except one of the lower. If we should wish to define the term "man," from materials supplied by the ordination lately given, either the attribute of animal life or that of organization would oftenest be the only one of the two in which we are directly interested. If the case be so, we shall content ourselves with asserting, either that "Man is an animal," or that "Man is an organized being." So, if we wish to divide the term "man," we shall almost always assert only, either that "Some men are Europeans," or that "Some men are Scotsmen:" we shall not make both assertions. In a word, we evolve only one step in the ordination; whether that be the first step either way from our given term, or a step more distant.

There are, however, three cases at least, all of them not only supposable, but actual, in which it becomes necessary to evolve more steps than one, or even a considerable number. In the first case, either definition or division is attempted, when our knowledge of the objects is so narrow as to yield only a series of terms, which do not justify a sufficient number of exclusions (co-ordinates being here the terms that will be wanting). Secondly, either may be attempted, when language does not furnish words clearly implicative of suppressed steps in a series. Lastly, either may be attempted, when, though knowledge and language should both be sufficient for their work, our definition or our division is designed to be the foundation of a very wide and complicated system of knowledge. Scientific definitions and divisions, for example, especially the former, are often necessarily complex, setting forth several steps from an ordinated series of terms; and the desire to simplify and abridge the series is one of the strongest of

those many reasons, which justify the invention of technical names.

55. The incompleteness of our examples of growing definition and division is a point lying much deeper than their redundancy. The reason of it, and the remedy, require to be considered with especial closeness in their bearing on the definition.

Definition at its third step of growth.

(1.) It is, as we have seen, necessary to a definition, that the term to be defined be placed in the extension of at least one other term. The objects denoted by the given term are thus included in a class, all the objects of which have a certain attribute; and this attribute is a mark of the given term, that is, a part of its comprehension.

In defining "man" from our series of terms, we must be able to predicate of it one of the superordinates in extension. We must at least be able to affirm, that "All men are animals:" and, for most definitions of the term, the wider affirmation, of "organized beings," will not be required.

(2.) Such a placing of the term in the extension of a superordinate, or even in that of several or many such, is not sufficient. By the hypothesis involved in ordination, there may be, and we know that in fact there always are, other terms, thinkable as co-ordinates of the given term. Of each of these co-ordinates the superordinate might be affirmed, as well as of the term to be defined. Therefore it is that, in our embryo definition, the superordinate term was undistributed: "All men—are—some animals;" there may be, and we know there are, other animals besides men.[1]

[1] Students of the science, who may be disposed to bestow close attention on the theory of the Definition, may be invited to scru-

(3.) What is sought, in addition to the superordinate, is, the means of distinguishing the given term from its co-ordinates. But distinction is negation. Therefore, besides affirming the superordinate, we must be able to deny all the co-ordinates. We must have, for incorporation, as a subordinate element of our predicate, the import of a proposition of exclusion.

Suppose we know only that there are animals which are not men. The two co-ordinate terms "men," and "not-men" (animals being implied), constitute together the immediate extension of the term "animal:" and either of the two is, by the law of predication for co-ordinates, deniable of the other. This filling up of the class "animal" by the two subclasses, would enable us to frame a definition, which could hardly ever be useful, but which might sometimes be the only one attainable, while in form it would be quite regular, though very awkward: "All men—are—all animals that are not Not-men."

But we may know something more: we may know names

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tinize for themselves the point which it is here attempted to bring out; namely, the function of co-ordination in the process of defining—the fact that one of the elements of the definition (it is that which the schools call the Specific Difference) is equivalent to a negation of the co-ordinates of the definitum. The embryo of this doctrine lurks in several systems of logic. But it does not seem to have anywhere come fairly above ground. Indeed, in some of the best of the German books, the difficulty (which has not been overlooked) of determining the relations between the definitum and the specific difference, has proved to be insurmountable. Twesten, the most clearly systematic among the formal logicians of Germany, has (strange to say) expressly denied the applicability of co-ordinates as elements of a definition. (*Die Logik*, p. 211.)

denoting all the co-ordinates of "man:" we may know that there are (according to a loose zoology, more generally understood than more scientific ones) five kinds of animals besides man. These five, taken together, become equivalent to our "animals that are not-men." Our definition will now stand thus: "All men—are—all animals that are not beasts, nor birds, nor fishes, nor reptiles, nor insects." But the definition, so altered, is still of little use. It cannot become extensively available, so long as it is merely negative of the co-ordinates.

(4.) The definition may be perfected when we have discovered some attribute, which is either possessed by the class denoted by the term to be defined, and wanting to all its co-ordinates, or possessed by all the latter, and wanting to the former. Such an attribute (in the former case), or its contradictory, denoting the want of it (in the latter), is a mark of the given term. The co-ordinates, as not possessing the mark, are thinkable as all of them contained in the contradictory of the given term, and may, therefore, be denied of it. The legitimacy of this denial is implied, when we affirm that the attribute, or its contradictory, is a mark of the term to be defined.

The schoolmen were wont to assign "rationality" as an attribute which is a mark of man, because alleged to be wanting to all other animals. "Not-rationality," the want of rationality, its contradictory, would thus be the attribute possessed by the co-ordinates, and wanting to man. Accepting this mark, we should now be able to express our definition in either of two shapes. Negatively, we should say, "All men—are—all animals that are not non-rational;" and here "non-rational" takes the place, and is an exact equivalent, of our "not-men," and "neither beasts nor" other

animals. Affirmatively, we should say, with exact identity of meaning: "All men—are—all animals that are rational."

It has been necessary, for the completion of the analysis, to bring out the negative form of the definition. But the affirmative form is always, and rightly, preferred when it is attainable: the distinctive mark is more readily useable in this shape. Besides this, it does more frequently offer itself in this shape than in the negative. It is usually easier to discover, by observation, an attribute possessed by one class and wanting in others, than to discover an attribute which, while wanting in one class, can peremptorily be asserted to be universal in each of several others.

(5.) In respect of formation, as it thus appears, a definition grows out of two several assertions. These are, both of them, in extension, though in different degrees; and, further, they are opposed in quality. It has been affirmed that the definitum (the term defined), is identical with a part of the extension of a superordinate term: it has been denied that the definitum is identical with the extension of any co-ordinate terms.

(6.) In result, a definition is, in the form it commonly wears, a predication in extension; because the term given to be defined tends naturally to preserve its place as subject. But, as its terms, being equivalents, are interchangeable, conversion throws it into comprehension. Which of the forms it may take, is a question utterly indifferent. For it is an affirmation that the *Definitum* and the *Definitio*, the subject and the predicate, are terms identical both in extension and in comprehension; that they are merely two several names for one and the same class of objects.

It is, however, the comprehension only that the definition evolves: the defined term being one of the two terms,

the other term explicates those terms which constitute its comprehension. The proposition is an assertion that the comprehension of the term defined is constituted by certain attributes; that the comprehension of "man" is constituted by "animality" and "rationality." Looking to the other side, all that we are told is this: that the extension of "man" is constituted by all objects, whatever they may be, which possess both of those attributes.

Division at its third step of growth.

56. The way having been found to the removal of incompleteness in a definition, similar dealing with a division is much facilitated.

The term to be divided is the name of a class: it will be divided in one step, when we have affirmed of it the names of all the co-ordinate sub-classes which constitute, in one degree, the extension of that class. The divided class, and the aggregate of the sub-classes into which it is divided, are co-extensive. The objects which, when thought of in one group, are denoted by the one given term, are the same objects which, when thought of in several groups, are adequately denoted, all of them, by the enumeration of the terms we have evolved out of the given one. The subject and the predicate of a proposition enunciating the division, are but different names for one and the same aggregate of objects: therefore they are interchangeable; and the conversion of the proposition is free both ways.

So long as our terms were only the given term on the one side, and one, or some, of the terms subordinate in extension on the other, one of the terms (the term given) was necessarily undistributed. We had to say, before, "Some men—are—all Europeans;" or, "All Europeans—are—some men." But, in order to complete our division, we learn

that the class "man," when we divide it on the principle of local habitation, may be loosely distributed into five sub-classes. At length, therefore, by uniting the names of all those sub-classes to form one of our terms, we gain a proposition in which both terms are distributed. We say, "All men—are—all Europeans, all Asiatics, all Africans, all Americans, all Australasians;" or, "All Europeans, all Asiatics, all Africans, all Americans, all Australasians—are—all men."

These forms of expression, however, are awkward, and may be deceptive. If they are to be adopted, the "all" will be understood (naturally, and perhaps unavoidably), not distributively, but collectively. On this footing the terms are equivalent to singulars; and the propositions are unmanageable. But let our "all" be understood distributively: we are thus led to the alternative or disjunctive form of speech in the enumeration of the subordinate terms. "All men—are—all men who are either Europeans, or Asiatics, or Africans, or Americans, or Australasians;" or, "Every several man—is—every several man who is either an European, or an Asiatic, or an African, or an American, or an Australasian:" and so when the terms are reversed.

Now, it is in this alternative shape, though without signature of the predicate, that we do always, in ordinary thought and speech, express a division. We say: "Every man is either a European, an Asiatic, an African, an American, or an Australasian;" or, "Every one who is either a European, an Asiatic, an African, an American, or an Australasian, is a man." This fact is a guide-post, pointing out the road by which we reach the application of divisions in reasoning.

Divisions may, of course, be carried down in more steps

than one; in as many steps, indeed, as our presupposed ordination allows, and the purpose of the division makes to be desirable. But, cumbrous even when embracing one step only, they become, when stretched farther, almost inexpressible in the shape of explicit propositions. In such cases, and sometimes in the simpler ones, they are usually left unexplicated. Scientific writers, especially in the sciences of Classification (where both definitions and divisions are often exceedingly complex), content themselves with exhibiting the ordinated series of terms in a tabular shape: and from this series special propositions are extricable when called for.

A division, then, is a proposition which, when the term to be divided is taken in its natural function as subject, is a predication in comprehension, but which is readily transformable into a predication in extension. It is the extension only of the divided term that is evolved. The divided term being one of the terms, the other explicates the terms which constitute its extension. The proposition is an assertion that the extension of the divided term is constituted by certain sub-classes; that the extension of "man" is constituted by all the objects of the five named classes. On the other side, we are told only this: that the comprehension of "man" is constituted by all the attributes, whatever they may be, which are possessed by all those objects.

57. The theory of division is not yet wholly before us. It is completed when we contrast the process with definition. The result rests, as closely as that of definition, on preformed propositions. The character of these is unchanged; but their relative prominence is reversed.

Division compared with definition.

(1.) There is, in division, a mutual negation of two co-ordinates; and this negation has for its basis the necessary inconsistency between a term and its contradictory.

Of the evolved terms constituting the extension of the given term, we fix our attention on some one: if we take more than one, these are thought as one. We must be able to think of this one term, and of all the others taken together, as being contradictories of each other. If "European" is the term we attend to, the other four are for us equivalent to "Not-European." Our implied negation is, that Europeans on the one side, and all its co-ördinates on the other, are mutually exclusive; that Europeans are not any persons who are either Asiatics or persons of any of the other classes. But this is on the assumption that the same negation might be made through the contradictories; that we should be expressing the same denial, although with a narrower assumed knowledge, by saying, that "Europeans are not Non-Europeans."

Each of the co-ordinates must thus, in its turn, be thought as deniable of all the rest, or as having all these as constituting its contradictory. If it were not so, there would be a manifest confusion of identities.

(2.) The negation of co-ordinates, which is left as implied in the definition, is, in the division, the element which is explicitly set forth. The thorough-going exclusion of the evolved terms from each other, is expressly signified by the alternative words "either" and "or."

(3.) There is affirmation with two terms, a super-ordinate and a subordinate. And, in division, though not in definition, the affirmation is at least double, and may be regarded as being often manifold.

There is presupposed the inclusion of all the evolved

terms in the extension of the term given to be divided: "Europeans are men;" "Asiatics are men:" and so on. This is self-evident as to all the positive terms. Each of our evolved terms, then is, virtually, "Men who are Europeans, Asiatics," and so on. But, though we were to pass only from "Europeans" to its contradictory, there would be the same implication: and this view brings out the point with especial distinctness. We must here have the same inclusion: "All Non-Europeans are men:" the contradictory term is, virtually, "men who are Non-Europeans." If the contradictory of "Europeans" were taken without this limitation, it must denote all thinkable objects besides Europeans: it would cease to be truly co-ordinate with Europeans; and the foundation of the division would be overthrown.

In a word, the contradiction, and consequent exclusion, which are thought in the process of dividing, are a contradiction and exclusion not absolute or pure, but only within the sphere or extension of the given term. The objects thought as constituting that sphere, are posited or assumed in the whole process: they constitute what has aptly (though not with this reference) been called the "Universe" of all the propositions which the division either expresses or implies.[1] The terms contradictory of each other, whether explicitly or virtually, are not pure contradictories, but only contradictories within the given sphere.

(4.) The affirmation of inclusion, which, in the definition, is the element explicitly set forth, is, in the division, the element which is left as implied. It lies, indeed, so deeply hidden, that logicians have sometimes overlooked it: a decisive instance is described in the next section.

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[1] De Morgan, *Formal Logic*, p. 38.

Division by dichotomy.

58. Every division, however complex, is thus reducible, at each of its steps, to a Dichotomy; that is, to the division of a class into two sub-classes opposed to each other by contradiction. The term X, if divisible positively by several terms, of which Y is one, is divisible also by the terms Y and Not-Y.

Dichotomy is not only the normal form of division, the form in which the primary principle appears most clearly. It is also a form of division which has practical uses, and which has, by some thinkers, been adopted as the explicit basis of all classification.[1] Requiring no positive assumption as to any of the co-ordinates but one, and regarding all the others as merely contradictory of the first, it has been vaunted as the ideal of a process fulfilling the requirements of a pure or *a priori* logic. In confutation of this claim it has been alleged, that the negative co-ordinate is not a pure contradictory of the positive: that when the class X is supposed to be divided into the sub-classes Y and Not-Y, the second sub-class is really "Those X's which are Not-Y's."

The correct view seems to be that, of which an explanation was attempted in the last section. It is true that *both* of the co-ordinate terms are positively limited, each of them being in thought included in the superordinate: the members which really divide the class X are these: "The X's which are Y's;" "The X's which are Not-Y's." The contradiction of terms which is gained, a formal and

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[1] It is enough to instance Peter Ramus among older thinkers, Jeremy Bentham among moderns. The latter name is certainly a symptom that dichotomy cannot be without its practical uses. As to the theoretical difficulties attending it, see especially Trendelenburg.

direct contradiction within the sphere of the term given to be divided, is the only contradiction which the case admits ; and, within the sphere thought of, it is a pure contradiction.

Even so considered, the facts refute the claim of dichotomy to being an application of the laws of pure thought, without any consideration of matter. But this, as we have seen, is a claim that cannot validly be urged in behalf of any logical law whatever. Least of all is it tenable in regard to laws which, assuming concepts and common terms as given, must presuppose some of the widest and most perplexing of the objective relations, under which only actual knowledge is possible.[1]

59. In all those logical systems which found their theory The five predicables.

[1] The dichotomous division has its chief value in the earlier progress of a science : there it is an admirable and often a decisive test. The principle of it, and often its form, enter widely into those processes of applied logic which are described as processes or methods of induction. But its direct use goes no further than allowing us to throw aside, by repeated exclusions, classes of objects which observation has shown to be alien from the purpose of our inquiry. It thus narrows, by successive steps, the ground over which our new observations have to be carried. We commence our scrutiny of the class X, by distributing it into two subclasses. The one of these is Y, of whose laws or attributes we know something : the other is Not-Y, in regard to which, as yet, we may know nothing. If Y does not satisfy the conditions of our problem, both Y and the containing class X are dismissed from our thoughts. Our field lies now in the class Not-Y ; and it may similarly yield Z and Not-Z, to be dealt with as before. This is one of the uses of the process ; but its variability, as a groundwork of exclusion, is very great.

of definition and division on the scholastic and Aristotelian opinions, that theory rests, for both processes, on the scheme of the Predicables. The doctrine of the predicables is, in some of its parts, clear and valuable: in others it is difficult alike of explanation and of application. So much of it must here be described, as shall exhibit the bearing of the analysis above proposed on the common rules of definition and division.

Predicables are terms affirmable, as predicates, of other terms. Further, the identical affirmation of singulars being neglected, all predicables are said to be common terms. All the common terms which are affirmatively predicable of others, must import, relatively to the subject of the proposition, one or another of five things. The Predicables are five:—Genus, Species, Difference, Property, and Accident.

(1.) Of any term given as subject, we may affirm, as predicate, its genus. The genus is the widest class in which, according to the view supposed in a given process of thought, the subject can be held as included.

(2.) Of the subject we may affirm its species. The species is any one of several narrower classes, actual or thinkable, which together make up the genus or widest class. In it, as in the genus, the subject is presupposed to be included.

(3.) Of the subject we may affirm a difference. The idea attached to this term has been defined and limited very variously. It receives, probably, the fullest justice when we say, that a difference is an attribute possessed by a whole class, and by that class alone,—that it is an attribute possessed by all the objects of a class, and not by any other objects. It is an attribute universal and peculiar to the objects of a class. Since there are two classes, a more ex-

tensive and a less extensive (the genus and the species), in either of which the subject may be included, the difference may, correspondingly, be of two several kinds. A generic difference is an attribute universal and peculiar to a genus, and thus distinguishing the genus from all other possible genera: a specific difference is related in the same way to a species. The former is of little or no use.

(4.) Of the subject we may affirm a property. This term, which has been described as variously as the difference, may be explained thus. A property is an attribute possessed by a whole class, but not by that class alone: it is an attribute possessed by all the objects of a class, but possessed also by other objects; it is an attribute universal but not peculiar to the objects of a class. Property, like difference, might be either generic or specific: but property is plainly useless both for definition and for division, unless in the way of preparatory exclusion; therefore the distinction has not been worked out. When the name is applied at all, property seems always to be held specific.

(5.) Of the subject we may affirm an accident. An accident may be said to be an attribute which is possessed by some of the objects of a class, but is not thought of as possessed by all of them. As we are here touching on individuality, accident is always held as specific; the species being, in this scheme, the lowest class, between which and the individual objects no class is thought as intervening. Accidents are further said to be either separable or inseparable. An inseparable accident is an attribute which we cannot, a separable accident is an attribute which we can, think of the subject as not possessing.[1]

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[1] In regard, here, to difference and property, and also in the

'he uses of he predicables in efinition nd division.

60. It has often been made a ground of objection to the predicables, that the criteria by which they are distinguished from each other presuppose an objective certainty, an insight into the true nature of the objects compared, which is alike impossible of attainment, and beyond the range of logical scrutiny. The charge is good against not a few of the explanations that have been given, especially of the last three. But the essential character of the scheme is quite in accordance with the fact, that all classification is merely relative, that the placing of objects in classes is nothing more than an operation of thought. The scheme, likewise, is easily useable, within proper logical bounds, as a means of explicating the results of a classification which has been thought out, whether in consonance, or in repugnance, to the real character of the objects.

This much having been premised, we shall readily perceive the bearings of the scheme on definition and division.

(1.) It is observable, in the first place, that the scheme supposes an ordination, of classes or common terms, embracing only two steps, a superordinate and a subordinate,—the genus and the species. It is admitted that more are frequently required; and the terminology has been tortured into elasticity, to make it answer the demand as fully as possible. Genus and species, we are warned, are relative terms: every class is a species in reference to a more extensive class; every class is a genus in reference

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next section, obligations are due to an *Examination of some Passages in Dr Whately's Logic*, by George Cornewall Lewis, 1829. Other recent writers, also, in England, have speculated much and acutely on the doctrine of the predicables. They are scrutinized very closely in Mansel's edition of *Aldrich*.

to a class less extensive. We are allowed to speak of a *summum genus*, and of subaltern genera contained in it; and we receive from some quarters a license to introduce sub-species.

(2.) Still, the range of terminology is palpably inadequate to many scientific purposes. It is especially so for those sciences, which have to distribute and redistribute, by inclusions and exclusions multifariously repeated, a vast number of known objects, related to each other by many inosculating points of resemblance and difference. For division in such cases, the scheme is impotent: it is weak for definition, unless where the objects to be defined have already had their leading relations thoroughly ascertained. In the physical sciences, particularly those dealing with organic bodies, animate or inanimate, special schemes have been constructed to meet the special claims. Indeed, there has lingered, in the technical nomenclature of modern science, hardly more than one little fragment of the Greek and mediæval structure. Genus and species, the two inherited names which alone keep their places, are used, by preference, to signify such classes as are characterized by firmly-marked points of resemblance and difference, and held, on strong grounds, to be actually related to each other by immediate ordination.

(3.) When the relations of objects have been precisely fixed, the scholastic scheme is perfectly fitted for explicating our knowledge of them in a definition. Accordingly, it is a received point of logical phraseology, that those definitions only which admit of being referred to the table of the predicables, are to be regarded as properly and strictly definitions. Such as are not so referable are by logicians usually called "descriptions."

A Definition proper, then, is a proposition defining a species: of the species it affirms its genus and its specific difference. For a definition of "man," "animal" may be taken as the genus; as the name of a more extensive class containing the species "man," and also other species. "Rational" may be accepted as the specific difference; as the mark which distinguishes man from other species of animals. We thus gain the definition: "Man is an animal rational;" the formation of which, from the ordinated terms, we have already endeavoured to watch.

Into the regular definition, then, there enter the first three of the predicables: the species, yielding the subject, the term to be defined; the genus, and the specific difference, yielding the terms constituting the predicate, which is the defining term.

(4.) Neither for definition, nor for division, do the last two of the predicables offer any materials.[1]

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[1] The Accident is properly affirmable of individuals only. If it is affirmable of more individuals than one, and if, in respect of it, these individuals are to be compared, they thus come to be thought of as a class (named or unnamed); and the accident becomes a difference, a species, or a genus. The accident, *qua* accident, is no element, no part, either in the extension or in the comprehension of the genus or the species, the only two classes whose formation the scheme presupposes. "Alexander is a soldier;" but his being so is merely an accident, an unimportant attribute, so long as he is considered merely as a man or rational being. "Alexander, and a good many others, are soldiers; therefore they are brave men." Here there is assumed a new reference to classes, in virtue of which "soldiers" has become the name of a class, which contains the class constituted by the named individuals, and is in its turn contained in the class "brave men."

The avowed difficulty, again, of determining, whether a given

61. The view which has here been described,—that of regarding definitions as being strictly and necessarily evolutions of the comprehension, divisions as being strictly and necessarily evolutions of the extension, of the term defined or divided,—does seem to furnish, and to be the only view capable of furnishing, reasons, scientific and universally valid, on which to rest the received formulæ and rules of both processes.

The logical foundation of definition and division.

The rules, indeed, are manifest corollaries from those doctrines, which it has been endeavoured to exhibit as governing the development of each whole of a common term.

Whether either a definition or a division be a true

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accident is separable or inseparable, must be solved by considerations which are entirely extra-logical.

The position of Property is essentially the same as that of accident, yet with an instructive difference. Accident not entering at all into our pre-formation of the genus and species, we had to travel quite aside from the given series of terms in order to make it logically available. Property does enter by implication into our idea of the species, and through it into our idea of the genus. Thus, in reference to the species and genus which yielded our definition of "man," organization is a property of "man." It did not aid our definition, because it is possessed by other objects besides man; but it is implied both in the species and in the genus: men are organized beings; so are animals. If, then, we wish to make "organization" available for definition, what we have to do is, not to desert the ordination given, but to carry it upward till it culminates in a wider class. By taking one step, we transform the property into a genus, available as part of a defining term: "All animals are organized beings possessing sensitive life." By taking two steps, we transform the property into a species requiring to be defined: "All organized beings are created things, having parts which operate on each other."

statement of the relations of the objects which the terms denote, is a question which logic cannot answer. All definitions and all divisions are, for logic, hypothetical merely: they are explicit assertions of objective relations; but these are such only as are presupposed in the meaning of the terms. Now, these relations are not exactly discoverable, unless through a distinct apprehension of the classification or ordination of the terms which are to appear in the proposition, as term defined, and as terms whose combination is to constitute the term defining. If a definition is to be framed, an ordination of the terms is the best preparation that can be made for it. If a definition is to be tested, the explicating of the ordination which it implies will be the readiest means of determining its value.[1]

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[1] Logical division, the exhaustive enumeration of the subclasses constituting a given class, cannot well be confounded with real division, or partition, the separation of an integral whole into its parts. Whether, again, definition by genus and species should be called "real" or "nominal," is a question which has been answered both ways; because different logicians have attached different meanings to the epithets. Sir W. Hamilton calls logical definition "notional;" and the same specific name may be given to logical division, if there should seem to be any risk of mistake.

The following are the rules for both processes, which, given by Aldrich, reappear with variations in most of our standard English books. (Mansel's *Aldrich*, pp. 30, 35):—

I. DEFINITION.

"(1.) Let the definition be *adequate;* otherwise it does not explain the definitum. For that definition which is more limited than the definitum, explains only a part, whereas the definitum is a whole: a definition which is more extensive explains a whole, whereof the definitum is only a part. (2.) Let the definition be *of*

itself clearer than the name defined. I say, of itself, *per se*, because, *per accidens*, that may be less understood which is better known by its own nature. (3.) Let the definition be expressed in a just number of proper words (words not figurative); for, from metaphors arises ambiguity, from too much brevity arises obscurity, and from prolixity arises confusion."

II. DIVISION.

"(1.) Let the dividentia or dividing members, severally, contain less, that is, signify less" (that is, let each of them be less extensive) "than the divisum or whole divided; for the whole is greater than the several parts. (2.) Let the dividing members, conjointly, contain neither more nor less than the whole divided; for the whole is equal to all its parts. (3.) Let the dividing members be opposite, that is, not contained in each other; for, without distinction, partition is fruitless."

PART THIRD.

THE DOCTRINE OF INFERENCE.

CHAPTER I.

The Character and Kinds of Inference.

The character of inference.

62. The scholastic logicians described the science as analyzing the products of three mental operations, specifically different: Apprehension, Judgment, and Reasoning. There is a psychological difference between the first two of these, the difference between thought unevolved and thought evolved: and the two kinds of facts yield products differing in form. It is now allowed, generally and rightly, that there is no such difference between judgment and reasoning: the latter operation is constituted by repetitions of the former. Whether we judge or reason, we are alike explicating, in forms yielding propositions, implied relations of given ideas and objects.[1]

[1] "According to these definitions [Locke's], supposing the equality of two lines A and B to be perceived immediately in consequence of their coincidence, the judgment of the mind is intuitive: supposing A to coincide with B, and B with C, the relation between A and C is perceived by reasoning. This is certainly not agree-

The forms, however, in which the data may be presented, differ so far as to modify secondarily the forms of explication, and especially by causing diversities in the degree of complexity. It is, therefore, desirable that the most prominent of the explicative forms should be studied separately. There is thus a practical reason for logically treating judgment and predication apart from reasoning or inference, and also for considering severally the leading varieties in the forms of inference.

The only formal difference which can enable us to distinguish, consistently and firmly, between predication and inference, is that which arises out of the distinction between apprehension and judgment. A process in which a proposition is evolved directly from given terms, is a mere predication. Every process in which a proposition is evolved directly from one or more given propositions, must be considered as an inference.

able to common language. The truth of mathematical axioms has always been supposed to be intuitively obvious: and the first of these affirms, that, if A be equal to B, and B to C, A and C are equal. Admitting, however, Locke's definition to be just, it might easily be shown, that the faculty which perceives the relation between A and C, is the same with the faculty which perceives the relation between A and B and between B and C. When the relation of equality between A and B has once been perceived, A and B become different names for the same thing. That the power of reasoning (or, as it has been sometimes called, the Discursive Faculty), is implied in the powers of intuition and memory, appears also from an examination of the structure of syllogisms. It is impossible to conceive an understanding so formed, as to perceive the truth of the major and minor propositions, and not to perceive the truth of the conclusion." (Dugald Stewart, *Outlines*, part i., sect. 9.)

The kinds of inference; immediate and mediate.

63. Every inference contains, in expression as in thought, two parts, that which is given and that which is sought,—the Antecedent and the Consequent. The immediate consequent must be one proposition only. But the antecedent may be either simple or complex: it may be constituted by one proposition only, or by more propositions than one.

An inference, whose antecedent is constituted by one proposition, is an Immediate Inference. There is explicated, in the antecedent, a relation between two terms: there is explicated, in the consequent, between the same two terms, another relation which had been implied in the given one.

An inference, whose antecedent is constituted by more propositions than one, is a Mediate Inference. The simplest case, that in which the antecedent propositions are two, is the Syllogism. The syllogism is the norm of all inferences whose antecedent is more complex; and all such inferences may, by those who think it worth while, be resolved into a series of syllogisms.[1]

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[1] By the older logicians, and by those of this country till recently, the name of Inference, Reasoning, Discourse (Shakspere's "*discourse of reason*"), was limited to the syllogism and processes yet more complex. The extension of it, so as to embrace the processes here called immediate inferences, seems to have been first made by some of the German logicians after Wolf: it was adopted by Kant, who has been followed in this point by all his countrymen. By more than one of these, however, it has been shown, that his distinction of the two kinds, as being respectively inferences of the understanding, and inferences of the reason, is not so much as consistent with his own psychology.

The secondary difference between inference and simple predication being recognized, the old limitation of the former name

By syllogistic inference we seek to explicate, in the consequent or conclusion, a relation between two terms. In the premises or antecedent propositions, there is not explicated any relation between those two terms; but in each of them there is explicated a relation between one of the two terms and a third. This third, or mediating term, stands so related to the other two, that the explicated relations between them and it imply a relation between the two themselves; and this is the relation which is explicated in the consequent.

In every process of inference, the consequent is the explicit assertion of an identity or non-identity between its two terms; which identity or non-identity was implied in the antecedent. When the predications are through common terms, the Quality of the consequent, as affirmative or negative, is determined, in the last resort, by reference to the ordination of the terms. The Quantity of each term of the consequent is limited by the quantity of each of the terms of the antecedent; and this quantity is traceable specially to the extension of each of those terms.

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not only is incorrect, but tends to disguise from us the real character of inferences which are immediate. Any valid reason for refusing the name to processes of this sort, would tell with equal force against the syllogism.

CHAPTER II.

Immediate Categorical Inference.

The modes of immediate inference; and their several characters.

64. All processes of immediate inference, whose validity is determinable by rules purely formal, may be embraced under four kinds. We infer immediately, either by Contraposition, by Subalternation, by Opposition (proper), or by Conversion.

These several kinds of processes stand towards each other in different relations of likeness and unlikeness. They may advantageously be compared from two different points of view.

(1.) The terms being common terms, each proposition, both antecedent and consequent, must be a predication of the subject, either in (or out of) the extension of the predicate, or in (or out of) its comprehension. If the matter of the assertions is not known, the data are not wide enough to indicate in which of the wholes the predication is. But certain points are ascertainable without interpretation of the terms.

In their relation to the two wholes, the first three processes are unlike the fourth. In inference by Contraposition, Subalternation, and Opposition, the antecedent and the consequent predicate in the same whole: both predicate either in extension or in comprehension. In inference by Conversion, the antecedent being a predication in one whole, the consequent is a predication in the other: the process consists, as has already been alleged, in the transference of

predication from extension into comprehension, or contrariwise.

(2.) The processes fall into other groups, when we consider the relation of truth or falsehood between antecedent and consequent. In this respect the first, second, and fourth kinds are unlike the third.

Contraposition, Subalternation, and Conversion, yield consequents, whose truth or falsehood, when it is determinable, agrees with the truth or falsehood of the antecedent. If the antecedent is admitted as true, the consequent must be admitted: if the antecedent is denied, the consequent must be denied. Opposition (proper) yields consequents, whose truth or falsehood, when it is determinable, is opposed to the truth or falsehood of the antecedent. If the antecedent is admitted, the consequent must be denied: if the antecedent is denied, the consequent must be admitted.[1]

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[1] Those who refuse to these processes the name of Inference, rest on this allegation; that, since the terms of the antecedent and those of the consequent are the same, the two propositions must merely express the same thought in two different forms.—Of Conversion this is plainly not true. It is far from being a matter of indifference to the real character of a judgment, which of its terms is taken as subject, and which as predicate: so much is evident without reference to the wholes of the concept, the examination of which founds more deeply the reasons of the difference. As to Opposition proper, the case is perhaps still clearer. We cannot be said to express the same judgment, in enunciating one proposition which is true, and another which, however closely related to the former, must be false. In respect to Subalternation, the question is narrower, lying merely between the "all" and the "some" of the subject. But here, likewise, the doubt falls away, when we remember that, on the strict analysis which we are bound to aim at, that which is really either subject or predicate, is not a common term which is distributable,

Inference by contraposition.

65. By Contraposition we gain a consequent, which must be admitted if the antecedent is admitted, and denied if the antecedent is denied.

We shall call the antecedent the Contraponend, the consequent the Contraposita.[1]

The process consists in transforming, through the law of non-contradiction, a given Affirmative into a Negative, which is accepted as equivalent or equipollent, or a given Negative into an equivalent Affirmative. In both cases the method is, to substitute for the predicate the term which is its contradictory, and then, as a necessary consequence, to change the character of the copula.[2] The principle is self-evident: what is done is to apply one of the logical axioms in its simplest shape. If, of a given term, we can affirm another, we must be entitled, of the first term, to deny a term which is contradictory of that other: if, of a given term, we can deny another, we must be entitled, of the first term, to affirm the contradictory of that other. If the X's are contained in the sphere of the Y's, they must be excluded from

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but that term peremptorily fixed as being either distributed or undistributed. "All X's," and "some X's," are the names of two several sets of objects. If there be any of the immediate inferences whose claim to the inferential character is reasonably doubtful, it is Contraposition, to which we now pass.

[1] The caution must be given that, in several of our English books of logic, the name of contraposition is given, not to this process, but to a twofold one, in which there really take place, first, contraposition; secondly, conversion of the contraposita. We shall speak of this complex process as Conversion through Contraposition.

[2] The *subject*, as denoting the notion or object given to be determined, must remain unchanged. It cannot be displaced by its contradictory, until it has first, by conversion, become *predicate*.

the whole of the sphere of the objects which are Not-Y's: if the X's are excluded from the sphere of the Y's, they must be contained in the sphere of the objects which are Not-Y's.

The affirmative becomes a negative, when, instead of affirming the predicate, we deny its contradictory. Thus, "All X's are some Y's" (A), becomes "The X's are not any Not-Y's" (E). The negative becomes affirmative, when, instead of denying the predicate, we affirm its contradictory. Thus, "Some X's are not any Y's" (O), becomes "Some X's are some Not-Y's" (I). There is in this way a possibility of contraposition, in both directions, between A and E, between I and O.[1]

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[1] This is a process which serves so many uses in the analysis of the syllogism, that it demands particular notice. The doubt, however, as to its claim to being held a genuine inference, is raised at once by some of the phrases which have just been applied to it. Two propositions, strictly and absolutely equivalent, cannot but be mere varieties of expression for one and the same judgment. But, on the other hand, it is questionable whether any two propositions do stand in such a relation. The minute anatomy of thought would exhibit fine differences in the character of the acts, even between cases of equivalence through synonymous terms, or through other variations not logically cognizable. One judgment is not in all points necessarily identical with another, though the two compare the same objects: the identity fails, as soon as there creeps in the slightest discrepancy between the relations in which the objects are thought.

It is fairly maintainable, that the contraponend and the contraposita are not absolutely equivalent,—that each of them brings out distinctly an element of thought which is merely implied in the other. I have, it may indeed be said, the same thought, a thought constituted by the very same factors, when I place X somewhere or other in the positive and limited sphere of the Y's, and when I exclude it from all points of the negative and undetermined sphere of the Not-Y's.

The kinds of opposition as commonly described.

66. In a large majority of logical systems, the name of Opposition is so applied, as to include Subalternation along with those other three relations to which, here, the name is

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Thus much may be admitted, that the two thoughts grasp one and the same relation of the objects. But they apprehend it from two opposite points of view. Subjectively or psychologically, it is not the same act of thought that places an object in one sphere and out of another. Objectively, again, or with reference to the products of the acts, the reality of a difference is made probable, if not absolutely certain, when we attempt using the one proposition or the other, alternatively, as a premise in a syllogism. Each of the two places the terms in a certain relation, not yielded by the other, to the other terms of the argument. Sometimes, therefore, the one proposition enables us to construct a good argument, while the other would generate a bad one; at other times the argument admits only one fixed form, if the one proposition is adopted, but is made flexible through the substitution of the other.

The question, as to the true relation between the contraponend and the contraposita, was pressed on modern logicians by Kant's "Categories of the Understanding." He recognized, in respect of quality, not only the affirmative judgment (X—is—Y), and the negative (X—is not—Y); but also the limitative or infinite (X—is—Not-Y). The negatively-determined term, (as "not-man"), was admitted by Aristotle (*De Enunciatione, passim*): and from his name for it, ὄνομα ἀόριστον, Boethius, and after him the schoolmen, called it (too widely) an "infinite term."

The old writers, moreover, acknowledged the infinite term as a *datum*, not for the predicate only, but also for the *subject*. When such a term does become the subject, the proposition has a peculiar character: it represents the explicit assertion of the ordinary "exceptives." "All things except the X's are Y's," gives, directly, "All not-X's are Y's."

It is curious to mark those ancient forms re-appearing, as data, in two recent systems.

1. Exceptive propositions give the foundation to Dr Boole's in-

limited. The three are these: Contrariety, Sub-contrariety, and Contradiction. The outline of the scheme, thus embracing all the four, may be used as an introduction to our separate examination of each.

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genious method of resolving (as others have attempted to resolve otherwise) all assertion into affirmation. His formula, "Y = X − Z," is interpretable as "The Y's—are—those X's which are not Z's." If, then, it is presupposed (as his notation postulates), that X is a genus containing the two species Y and Z, the assertion may take this form: "The Y's—are—Not-Z's;" which is equivalent to denying a term of its co-ordinate.

2. The infinite term, again, yields the characteristic forms to the scheme of predication worked out by Professor De Morgan, through the terms which he inconveniently calls contraries (*i.e.*, contradictories in the received nomenclature, as X and Not-X). Admitting "infinites," both as subjects and as predicates, he gains, as data for inference, eight "standard varieties of assertion," all treatable as A, E, I, O. The first four have, as subjects, positive terms (X): the last four have infinite subjects (not-X), and are virtually exceptives. Mr De Morgan is perfectly correct in deriving from each of his eight leading propositions two others, which thus make up his "contranominal" forms of predication to twenty-four. Only, not distinguishing between conversion proper and conversion through contraposition, he leaves in implication, in each of his deductions, one step, which, if supplied, would enable us to make all his inferences through the received rules. The omitted step is always a simple converse, from which his second consequent is deducible: for A and O it is the converse of the first consequent, for E and I the converse of the given antecedent. His first and eighth forms will illustrate both cases. 1. "All X's—are—Y's (A)=*Contraposita:* The X's—are not—Not-Y's (E)=*Converse:* The Not-Y's—are not —X's (E)= *Contraposita:* All Not-Y's—are—Not-X's (A)." 2. "Some Not-X's—are—Not-Y's (I)= *Contraposita:* Some Not X's —are not—Y's (O)=*Converse of the I:* Some Not-Y's—are—Not-

No propositional forms but A, E, I, and O, being admitted, we can, with any two common terms, form four propositions only. Any two of these are said to be opposed to each other, in respect that they must differ either in quantity or in quality, or in both. The kinds of opposition thus appearing are four.

I. Propositions agreeing in quality, but differing in quantity, are called, in reference to each other, Subalterns. The universal is the Subalternant, the particular the Subalternate. Any two terms furnish two pairs of subalterns: A and I, E and O. The same laws govern both pairs: hence the relation has only one name.

II. Any two terms furnish also two pairs of propositions, agreeing in quantity, but differing in quality. The same laws do not govern both pairs: there are two relations, and hence two names. (1.) The two universals, A and E, are called Contraries. (2.) The two particulars, I and O, are called Subcontraries.

III. Propositions differing both in quantity and in quality are called Contradictories. Any two terms furnish two such pairs: A and O, E and I. There is here but one relation, and hence one name.

It is convenient to be thus enabled to look, at one glance, over all the possible combinations of the same two terms in assertions of inclusion and exclusion. The survey is usually facilitated, in the books, by the placing of the four symbolic letters in the angles of a square; the universals standing

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X's (I)=*Contraposita:* Some Not-Y's—are not—X's (O)." All Mr De Morgan's eight contranominals are set forth by Boethius, in his *Introductio in Syllogismos Categoricos* (*Opera*, ed. 1570, p. 570, and elsewhere).

above and the particulars below, affirmatives on the left hand and negatives on the right. The relations or affections of each two propositions are then expressible by names placed in the sides of the square and in its diagonals.[1]

But subalternation, yielding a consequent consistent with the antecedent, and the other relations, yielding consequents inconsistent with the antecedents, ought, as modes of inference, to be in some way distinguished from each other: and the name of opposition, aptly designating the last three, is hardly germain to the first.

The general character of inference by opposition proper.

67. The rules which determine the deducible truth or falsehood of the consequents gained through Opposition proper, are, for all the modes, so very obvious, that in most of our English treatises they are laid down without proof. But it is right to show, as briefly as may be, yet without leaps in argument, how they are traceable to the law of non-contradiction in one or more of its phases.

That which we seek to infer through opposition, is not a consequent consistent with the antecedent; not a consequent whose truth is involved in the truth, or its falsehood in the falsehood, of the antecedent. We seek a consequent inconsistent with the antecedent,—a consequent so related to

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[1] We very often speak of assertions which we hold to be contradictory of each other, as being "diametrically opposed." The phrase is one of many which have migrated into common life from the scholastic cloisters. Substitute, for the square in whose angles the symbolic letters are now usually placed, a circle described about it. The diagonals of the square become *diameters* of the circle; and the pairs of contradictories stand at their extremities.

the antecedent, that the truth of the latter shall involve the falsehood of the former, and the falsehood of the latter the truth of the former. In a word, our two propositions ought to be so related, that the laws of difference and excluded middle shall strike at them directly: they should be peremptorily and necessarily contradictory of each other; like the assertion, "X is Y," as compared with the assertion, "X is Not-Y."

But our subject, being a common term, may be either distributed or undistributed. This variability takes away the power of applying the two laws with the same simple universality, in which they govern propositions whose subjects are singulars. We have to take account of the identity and non-identity of classes and parts of classes, in all the modes of combination allowed by the four propositional forms. Having completed this inspection, we find that the contradiction between antecedent and consequent is not universally and formally guaranteed, unless when the two propositions have a maximum of difference, that is, unless when they differ both in quality and in quantity.

Accordingly, propositions thus related are called Contradictories by way of eminence. This kind of opposition leads always from affirmation to denial, and from denial to affirmation. Its rules, if first established, facilitate the proof of the rules governing the other two: Contrary opposition, which leads only from affirmation to denial; Subcontrary opposition, which leads only from denial to affirmation. Subalternation, which leads from affirmation to affirmation, or from denial to denial, will find its place afterwards, and complete our review of the relations connecting all propositions framed with the same subject and the same predicate.

Inference by contradictory opposition.

68. That relation of propositions which, as yielding the only peremptory inconsistency, is emphatically called Contradiction, subsists between A and O, and between E and I. Of any two contradictory propositions, the one must be true and the other false. If the antecedent is admitted, the consequent must be denied: if the antecedent is denied, the consequent must be admitted.

If we had to seek the contradictory of a given proposition, the problem would in effect be this: Given an assertion which is assumed to be either true or false; to find the narrowest assertion that would, in all possible instances, be inconsistent with the assumption.

The solution might be attained very easily through the laws which govern concepts. If we start from one of the universals as true, we assume that a whole class of objects have (or want) a certain attribute. We have thus a proposition either in A or in E. Evidently inconsistent with this would be the truth of the opposite universal (E or A), asserting that none of the objects have (or want) the attribute. But, if our first universal were assumed to be false, there would not be a necessary inconsistency between this assumption and the truth of the opposite universal. Though we have denied that all the objects have (or want) the attribute, we may still be able either to affirm, or to deny, that none of them have (or want) it. A thorough-going inconsistency, therefore, does not subsist between the universals. But there is such an inconsistency between a universal and the opposite particular. If it be true that all the objects have the attribute, it must be false that some of them have it not: if it be false that all the objects have it, it must be true that some of them have it not. The con-

tradiction keeps its hold, whether we take, as our antecedent, the universal or the particular.

The leaning of affirmation and negation on identity and difference, and the necessary determination of thought towards the one or the other, may be brought to light as affecting these results, by the scrutiny of an example. The A and O will suffice: "All X's are some Y's" (A); "Some X's are not any Y's" (O).

The A is interpretable thus: "All the objects we call X—are identical with—some of the objects we call Y." The O is thus interpretable: "Some of the objects we call X—are non-identical with—all the objects we call Y."

In the first place, both of these assertions cannot be true. If we are entitled to affirm that all the objects X are the same objects which (with others) we call Y, we are much within the mark of safety, when we deny that some of the objects X are different objects from all those which we call Y. If we are entitled to affirm that some of the objects X are different objects from all those we call Y, we cannot possibly affirm also, that all the objects X are the same objects with some of those we call Y. If both assertions were true, there would be some X's which are identical with some Y's, yet non-identical with any Y's. We should, in effect, have affirmed, of the same subject, two contradictory predicates, Y and Not-Y.

On the other hand, both of the assertions cannot be false. Every thinkable object must be either Y or Not-Y. All the X's must either be some of the Y's, that is, identical with some of the objects in the sphere of Y; or they must be some of the Not-Y's, that is, some of the objects which are beyond that sphere.

In short, the one of the two assertions must be true, the

other false. If the antecedent is given as true, we do, in other words, affirm the identity or non-identity of the objects designated by the terms: the consequent in this case involves a denial of that identity or non-identity. If the antecedent is given as false, we thus deny the identity or non-identity of the objects: and in this case the consequent involves an affirmation of that identity or non-identity.

69. The relation of Contrariety subsists between A and E. Of two contrary propositions, both cannot be true, but both may be false. If the antecedent is admitted, the consequent must be denied: but though the antecedent should be denied, the consequent is not therefore necessarily admitted.

Inference by contrary opposition.

(1.) The objects denoted by the term which is the subject are the same in the two propositions: "All the X's are Y's" (A); "the X's are not Y's" (E). If both were true, the two would coalesce into the one self-contradictory assertion, that the X's are both Y's and Not-Y's. Therefore, if either is true, the other must be false.

(2.) The assumption that A is false, amounts to this only: it is not true that all the objects of a class possess a certain attribute. But this leaves open either of two cases. First, it may be true that none of the objects of the class possess the attribute; that is, in other words, the E is true. Secondly, it may be true, that some of the objects do possess the attribute; that is, the I is true. But if the I is true, its contradictory must be false: and that contradictory is the E. The same proof would be applicable if we made the E our starting-point. Therefore, though one of the contraries is false, the other may be either false or true.

70. The relation of Subcontrariety subsists between I and

Inference by sub-contrary opposition.

O. Of two subcontrary propositions, both cannot be false, but both may be true. It the antecedent is denied, the consequent must be admitted: but though the antecedent should be admitted, the consequent is not therefore necessarily denied.

Our propositions are these: "Some X's are Y's" (I); "Some X's are not Y's" (O). Evidently there are here the narrowest possible grounds of determination. The interpretation of the sign of quantitative limitation must be narrowly looked to.

(1.) If the subject were quantitatively definite, it would signify, for each of the two propositions, "Certain X's;" "Those X's of which I now think." We should, on that supposition, have to demand an answer to the question; whether the X's thought of in the I are the same X's which are thought of in the O, or a different group of X's. If they are the same X's, the two propositions have the same subject, and are inconsistent. If the X's are different X's, the two propositions have different subjects; and an assertion in regard to the one subject determines nothing for an assertion in regard to the other. In effect, the term "Certain X's" is, as was noted in a preceding section, virtually equivalent to a common term distributed: the propositions are in substance universal, both of them, however, having, as subjects, terms which are ambiguous.

(2.) The subject being quantitatively indefinite, according to the orthodox logical interpretation of the sign, the case stands quite otherwise. The subject constitutes, of the objects denotable by the subject-term, a part which is in every direction indefinite: the part is some or other, a few or many, some and perhaps all, but without our being entitled positively to assume all.

First, then, let either of the propositions be assumed to be false. It is false that "Some X's are Y's" (I). But, by the law of excluded middle, all the X's, like all other thinkable things, must either be or not be Y's. Since, then, we have denied the assertion that some of them are Y's, we are driven on the assertion that some of them are not Y's. If the I is false, the O must be true. Starting from the O, we should reach the same result in regard to the I.—The demonstration may be made more exact, if we choose to anticipate the doctrine of subalternation. It is false that "Some X's are Y's" (I): therefore the contradictory of the I must be true; that is, it is true that "The X's (any) are not Y's" (E). Therefore the subalternate of the E must be true; that is, it is true that "Some of the X's are not Y's" (O).

Next, let either of the propositions be given as true. It is true that "Some X's are Y's" (I). Nothing is thus given as to the whole class X: for aught we know, all the X's may be Y's, or some of them may not be Y's. If "All the X's are Y's" (A), the O will be false, as being the contradictory of A: if "Some of the X's are not Y's," this is a direct assertion of the truth of the O.

Inference by subalternation.

71. The relation of Subalternation subsists between A and I, and between E and O. By subalternation we may infer, either from whole to part, or from part to whole; from subalternant to subalternate, or from subalternate to subalternant.

In inference from Subalternant to Subalternate, if the antecedent is admitted, the consequent must be admitted. In inference from Subalternate to Subalternant, if the antecedent is denied, the consequent must be denied.

The peremptory consequences go no further. If the subalternant, as antecedent, is denied, the subalternate, as consequent, is not therefore, necessarily, either denied or admitted. If the subalternate, as antecedent, is admitted, the subalternant, as consequent, is not therefore, necessarily, either admitted or denied.

Let our propositions be the affirmatives: "All the X's are Y's" (A); "Some of the X's are Y's" (I). None of the preceding modes of inference lean, so openly as this, on the laws of predication through ordinated terms. From these laws, indeed, the rules might be directly deduced. The "all X's" and "some X's," stand really in the relation of superordinate and subordinate. If the "some X's" had a name, as "all Z's," they would constitute a sub-class included in the class X; and, if the fact were so, all the cases both of affirmation and of denial would be regulated, directly, by the canons laid down for predication in extension. But subalternation stands on the hypothesis, that the class denoted by the subject has not been divided into subclasses; and, on this footing, the rules may be justified by an immediate appeal to the elementary and universal principles of predication.

(1.) Of two subaltern propositions, both may be true, or both false.

Formally, we cannot determine which of the alternatives holds. If all the objects denotable by the subject-term are denotable also by the predicate-term, the A is true; and consequently the I also is true, the particular sign having its usual logical meaning. If none of the objects denotable by the subject-term are denotable by the predicate-term, we should contradict this assertion by affirming the predicate of the subject either in whole or in part: A and I

would, both of them, be false. The negatives are readily determinable in the same way.

(2.) If the subalternant is true, the subalternate must be true: if the subalternant is false, the subalternate may be either false or true.

The first section of this rule follows from the character of the logical "some," of which we have just been reminded. This, the most obvious law of subalternation, is also that which is most widely applicable. But suppose A to be false: it is false that "All the X's are Y's." This assumption is consistent with the supposition that the E is true; and, if so, I, the contradictory of E, is false. It is consistent also with the supposition that the I is true: though it is not true that all the objects of a class have the attribute Y, it may be true that some of them have it.

(3.) If the subalternate is false, the subalternant must be false; if the subalternate is true, the subalternant may be either true or false.

On the one hand, suppose the subalternate I to be false. Then its contradictory E must be true; and A, the contrary of E, must be false. Or take it thus: By hypothesis, the I is false. Assume the A to be true; therefore all its subalternates are true, which is contradictory of the hypothesis: therefore, by the law of excluded middle, the A must be false.

On the other hand, suppose the subalternate I to be true. Having learned nothing as to the whole class of X's, we are at liberty to assert also that the A is true. Or, with equal right, we may assert that O, the subcontrary, is true; but, if so, the A, its contradictory, must be false.

72. By Conversion we gain a consequent, which must be

The received rules of inference by conversion.

admitted if the antecedent is admitted. When the conversion is thorough, the consequent must also be denied if the antecedent is denied. Thorough conversion is reciprocal. But a defect in the received method of converting A makes it an exception: the denial of A, as antecedent, does not enforce the denial of the proposition usually accepted as its consequent.

The common doctrine of Conversion may be explained as follows:—

Conversion of a proposition is the transposition of its terms. The antecedent is called the Convertend, or Exposita (the proposition set forth to be converted); the consequent is called the Converse (the given proposition converted). The formal rule is this: that no term which was undistributed in the convertend shall be distributed in the converse. The reason is plain. Conversion is an illative or inferential process: it aims, in the narrowest view, at deducing a proposition which must be true if the given proposition is true; and from an assertion of "some" given as true, we cannot deduce an assertion of "all" as true. The received propositional forms, A, E, I, O, being regarded as the only cognizable forms, the non-distribution of terms in some of these makes it impossible, that each of them shall yield a converse of the same form as the convertend. Accordingly, three methods of conversion are laid down, as applicable severally to the several forms: Simple Conversion; Conversion *Per Accidens;* Conversion by (properly through or after) Contraposition.

(1.) Simple Conversion is a mere transposition of the terms of the convertend, both quantity and quality remaining unchanged. We may thus convert E, which, distributing both terms, yields another E. The given predicate, being dis-

tributed, becomes legitimately the subject of a universal: the given subject, being distributed, becomes legitimately the predicate of a negative. Thus, also, we may convert I, which, distributing neither term, yields another I. The predicate, though undistributed, is legitimately usable as the subject of a particular: the subject, though undistributed, is legitimately usable as the predicate of an affirmative. Thus: "The X's are not any Y's," gives, "The Y's are not any X's." "Some X's are some Y's," gives, "Some Y's are some X's."

(2.) Conversion *per accidens* is a transposition of the terms of the convertend, without change of the quality, but with a limitation of the quantity from universal to particular. A is not convertible simply into an A, because its predicate is undistributed. But we may convert it, *per accidens*, into an I: its predicate, though undistributed, may become the subject of a particular. "All X's are some Y's," cannot become "All Y's are X's;" but it does give, "Some Y's are X's." Thus, also, it is said, we may convert E into O. But the process yielding the O is really double: its second step is an inference from subalternant to subalternate. "The X's are not Y's," gives, by simple conversion, "The Y's are not X's:" whence comes, by subalternation, "Some Y's are not X's." For E, indeed, the process is seldom, if ever, put to use.

(3.) Conversion through Contraposition is truly, like the conversion of E into O, a double process. From the convertend there is first inferred an equivalent *contraposita;* and then this contraposita is converted. This complexity must be exhibited, if we are to explain rightly the character of the process. Being usually required only for O and A, it is treated in most of the books with exclusive reference

to them. But it covers E likewise. (1.) O cannot be converted directly. For its subject, being undistributed, cannot become the predicate of a negative; and the attempt to infer an affirmative with the same terms, would be self-evidently absurd. But, the negative sign of the copula being transferred to the predicate, we have thus inferred, from the O, its contraposita, an equipollent I: "Some X's are not Y's," becomes "Some X's—are—(some) Not-Y's." This contraposita I is then simply converted into another I: "Some Not-Y's are X's." (2.) A, though convertible directly, *per accidens*, is also convertible through contraposition. We first contrapose, by substituting, for the affirmation of the predicate, the denial of its contradictory, which transforms A into E: "All X's are (some) Y's," gives, "The X's—are not—(any) Not-Y's." The contraposita E is then simply converted into E: "The Not-Y's—are not—(any) X's." (3.) E also is evidently so convertible: its contraposita is an A, the converse of which, *per accidens*, is an I. But, for E, no use is made of the process. (4.) I is evidently not so convertible: its contraposita would be an O; which does not admit direct conversion.[1]

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[1] The Rules of Conversion, by all its three methods, were symbolized by the schoolmen in two mnemonic lines, in which the vowels of the nominative words designate the forms A, E, I, O.

> "F*e*c*i* simpliciter convertitur; *e*v*a* per accid.;
> F*a*x*o* (or *a*st*o*) per contra.: sic fit conversio tota."

If, the antecedent being true, the consequent is therefore true, why do geometers prove both a theorem and its converse? Because the proposition which they (and all of us, sometimes, in common speech)

Some logicians, both ancient and modern, have denied, on insufficient grounds, the competency of conversion through contraposition. They hold O to be inconvertible; and, of course, they decline to use the indirect process for A. They thus narrow our power of dealing with the two most difficult of the syllogistic moods.

Systematization of the rules of conversion.

73. The received doctrine, when reduced thoroughly to a system, gives the following results:—

(1.) There are really no more than two methods of conversion: conversion simple, applicable to E and I; conversion *per accidens*, applicable to A. O is not convertible by either method.

(2.) Every proposition admits contraposition; and of every form, except I, the contraposita is convertible. O becomes thus convertible indirectly, but not otherwise: its contraposita, an I, may be converted simply. A and E also are convertible through contraposition: the contraposita of A, being an E, is convertible simply; the contraposita of E, being an A, is convertible *per accidens.*

(3.) Since the converse of E admits a subalternate, E is thus indirectly convertible into O.

(4.) The four received forms of propositions thus admit, either directly or indirectly, the following converses, all of which are currently recognized:—A yields directly I, indirectly E: E yields directly E, indirectly O: I yields directly I: O yields indirectly I. E yields also, indirectly, an I, not currently recognized.

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call a converse, is not a logical converse. It has not either term the same with either term of the proposition which is nominally its exposita.

The rule by which these processes are guarded, and the directions given for its use, are traceable upwards, by a very short resolution, to the law of non-contradiction, as brought to bear on predication through common terms. The objects denoted by the subject (of which the quantitative sign is an integral part), and the objects denoted by the predicate (the quantitative sign again considered), are thought as identical when the convertend is affirmative, as non-identical when the convertend is negative. If both convertend and converse affirm when the objects are thought as identical, and deny when the objects are thought as non-identical, each of the terms may be indifferently subject or predicate. The rule, as to distribution of terms, simply prohibits us from interpolating, through either term of the converse, assertion in regard to any objects not named in the convertend.

Supplement to the doctrine of conversion. 74. The strict application of the law of identity shows, at a glance, that the converses of E, I, and O, must be false if the convertends are false. It shows also that, and how, this consequence should, but, on the ordinary interpretation of I, does not, follow as to the converse of A.

Whenever "all" is given in the convertend, we are unquestionably entitled to "all" in the converse. We have a right to infer, in converting A, not I merely, but I^2. "All X's—are—some Y's," being an affirmation of the identity of subject and predicate, yields, lawfully, "Some Y's—are—all X's." If the A is denied, so must the I^2 be: if it is denied that "All men—are—(some) liars," it must be denied that "Some liars—are—all men (the only men)." But, I^2 being unacknowledged in the orthodox scheme, the recognized and only possible converse of A is I: and, mani-

festly, though it were denied that "All the X's are some Y's," this does not necessitate the denial that "Some Y's are some X's."

Is the received converse of A, then, logically incorrect? Not in the least. The case is only that, read as an I, it alleges less widely than it might: the fault, like every other in the current systems, lies on the side of safety.

The truth is, that, in the conversion of A into I, there lies hidden a process of subalternation. The A yields I^2 as its exhaustive converse, which is true if the convertend be true, but false if the convertend be false. The I, which is usually accepted as the converse of the A, is virtually a subalternate of this full converse: it is true, by the principle of subalternation, if its subalternant is true; but it may, by the same principle, be either true or false if its subalternant be false.[1]

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[1] This cryptic process may be brought to the surface without I^2, but still more readily through it.

(1.) Let both A and I be given: "All the X's are some Y's," and "Some X's are some Y's." The I, "Some Y's are some X's," which we are required to accept as the converse of the former, is the full and genuine converse of the latter. Surely it will not be maintained that the subalternant can yield no wider inference than the subalternate. Our process implies our having first inferred an I from A by subalternation, and then simply converted the I. It is thus, in fact, that the conversion of A into I is justified by Boethius. (*Opera*, p. 575.)

(2.) If I^2 is taken into account, there emerges a relation of the propositions, which is disguised by the imperfection of the quantitative signs, yet is exactly conformable to admitted logical laws. I is virtually a subalternate of I^2, and is therefore inferrible from it. Given I^2: "Some X's—are—all Y's (the only Y's);" we find I: "Some X's (fewer than the first 'some')—are—some Y's." If

The process of conversion has thus been considered from the common position, with the one exception of A. Its laws have been referred to the principle of non-contradiction, as it affects propositions regarded without the analytic

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a part of the X's constitute a whole class Y, then, plainly, a part of that part of the X's must constitute a part of the class Y. When we compare our two propositions, we discover that our indeterminate particularity, though still indeterminate, has shrunk in the higher limit of its dimensions. The "some X's" of our I are only "some of the some X's" of our I^2. It is true that "Some mortal creatures are all (the only) human beings:" and what follows, on the principle of subalternation, is, that "Some of those some mortal creatures are some human beings."

The question raised by the assumed falsehood of A is here equally easy of decision. There is no inconsistency in our holding I^2 to be false (as it must be, if the convertend A is so), and in yet finding it impossible to determine whether the subalternate I be false or true. Our denying an assertion, made as to the whole of the first and larger part of our X's, does not give the slightest reason for denying the same assertion as to a part of that part.

In that aspect of the case which was first presented, the conversion of A into I was alleged to imply a subalternation followed by a conversion: in the other aspect, it has been alleged to imply a conversion followed by a subalternation. The two views, though the latter more directly than the other, conduct us to the same result. It is demonstrable, on grounds purely logical, that, when we accept, as a conversion of A, its transformation into an I, the inference covers a part of the subject-class, which, although indeterminate, is yet smaller, possibly or actually, and must necessarily be thought as smaller, than the part as to which the inference might have been drawn. If, therefore, in a process of reasoning, an A is one of our steps, and if, requiring to convert it, we content ourselves with I, we are indeed safe as to the subsequent progress of the argument; but we have narrowed our data in a way which may force a narrowing of our ultimate result.

dissection of the wholes of the common term. So long as we do not seek to apply the process to any use beyond the determination of the consequent, no deeper analysis is required.

But, when we have to consider the bearing of conversion on the syllogism, it will become imperatively necessary to look at the process from a more commanding station. Its true character, as being a transference of predication from extension to comprehension, or contrariwise, will then come out with irresistible force of evidence.

Inferences from and to propositions of constitution.

75. In the preceding treatment of the doctrine of immediate inference, no predicative forms are accepted as data, except the received propositions of inclusion and exclusion, A, I, E, O. Nor has it been necessary to take account of any other forms, unless in showing that I^2 gives the only full expression for the converse of A.

If the two propositions of constitution, A^2 and I^2, are combined, first with each other, and afterwards with each of the four received forms, there appear nine pairs of predications, involving a new series of relations. These are singularly barren as grounds of immediate inference. The fact intimates, not only how seldom such assertions can enter into our ordinary trains of thinking; but likewise how little reason there is for hoping, that their incorporation with the received scheme would materially increase the applicabilities of logical science.

One striking point is this. No two of the new pairs of propositions would be formally contradictories: no two are so related, that the one must be true and the other false. We gain only relations corresponding to subalternation and contrariety, with one which resembles subcontrariety.

This supplementary scheme of inference, in short, is philosophically interesting rather than practically useful. The subjoined summary will indicate all the results that can here be dealt with.*

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* The details may be gleaned from the table given (with warning that it "may not be quite accurate in details"), by Hamilton, *Discussions*, Appendix, ii., p. 637. In that table, a distinct separation is made, between the relations arising out of the two interpretations of the limitative sign, as "some at least," and "some at most;" both of which Sir W. Hamilton desires to introduce into the science. Neither of the interpretations seems to be excluded by Mr Thomson in his "Tables of Opposition of Judgments." (*Laws of Thought*, ed. 1854, p. 197.) Notice has already been given, that, in the present treatise, the received interpretation, "some at least," is steadily adhered to: if there be any deviation, it is an oversight.

1. The following four pairs of propositions are virtually contraries: A and I^2, A^2 and E, A^2 and O, I^2 and E. On the assumption, that the "some" is "some at least," both cannot be true, but both may be false. We may infer, therefore, from the truth of either to the falsehood of the other, but not inversely.

2. On the same assumption, these two pairs are virtually subalterns: A^2 is subalternant, I is subalternate; I^2 is subalternant, I is subalternate. If the subalternant is true, the subalternate is true, and may be inferred from it. If "All X's are all Y's," it follows, that "Some X's are some Y's:" the quantity of both terms is expressly limited. If, again, "Some X's are all Y's," it follows, that "Some X's are some Y's." The quantity of the predicate is expressly limited, but that of the subject also is limited in reality: the "some X's" which are identical with "some Y's," are not thought as being co-extensive with, but as being possibly only a part of, the "some X's" which are identical with "all Y's."

In regard, however, to those six pairs of propositions, the opposite interpretation of the "some" would leave the relations unaffected. As to the other three pairs, the case stands quite otherwise.

3. In either view, I^2 and O approach the relation of subcontraries. (1.) If the "some" is "some at least," the two propositions stand thus:—If I^2 is true, O must be false; admitting I^2, therefore, we may inferentially deny O. But, if O is true, I^2 may be either true or false. And both may be false. (2.) If the "some" is "some at most," then, since this excludes "all," I^2 being true, O is inferentially true likewise. If it be assumed as true (see Hamilton, p. 636), that "Some dogs (but not all) are all animals that bark," it must follow, as true, that "Some dogs (but not all) are not any animals that bark."

4. The two pairs still to be considered are, A^2 and A, A^2 and I^2. The relations of both are troublesome. By Mr Thomson these pairs are described as "inconsistent," (that is, as affirmatives standing in the relation of contrariety). In Hamilton's table, the pairs are "inconsistent" on the assumption of "some at most:" on the other assumption, they are not marked at all; but neither is any inference stated as admissible from the one to the other. In other passages, however, Sir W. Hamilton seems to disallow absolutely the consistency of A^2 and A, from which doctrine would follow the inconsistency of the other pair.

(1.) If the "some" is "some at most," the pairs stand, plainly, in relations of contrariety; the causes of the inconsistency lying more or less deep according to the quantity of the terms. (2.) If the "some" is "some at least," it is not easy to discover sufficient reasons for refusing to classify the pairs with the other subalterns. For, in this view, in the first place, A is only an incomplete or cautious assertion of A^2, and may safely be inferred from it: if it is true that "All X's are all Y's," it must be true that "All X's are at least some of the Y's, and perhaps all of them." Or we might take the question thus: If the X's constitute the class Y, every individual X must be identical with some one or other of the Y's. This, perhaps, is the plainer case of the two. If, again, the limitation, which here falls on the predicate, were to be transferred to the subject, the A^2 would yield I^2. "Some at least, and perhaps all, the X's, are all Y's." But as to this derivative assertion (I^2), even though we should be satisfied that it sustains the formal test,

we cannot but see that it serves no use. The limitation of the predicate had given us, in the A, an assertion easily thinkable as contained under the admitted A^2: but the limitation of the subject is virtually a thinking away of our A^2, and the substituting, in its place, of a judgment which leaves the A^2 as doubted.

This glance towards the practical side, suggests yet a wider consideration. Using words in their ordinary meanings, no one would dream of inferring, from A^2, either A or I^2. But why? Because our spontaneous "some" is always "some at most, some not all." In any use, therefore, which is not guarded by technical rules, the pairs would do duty as contraries.

Still, it must be added (not without reluctance), the more cautious interpretation of the "some" is, in the first place, the only one that can be brought to bear on the received logical system. That system falls to pieces as soon as the other interpretation is let in. Inference, for instance, from subalternant to subalternate, with all its syllogistic applications, requires the former as a foundation. Again, it has not yet been made unquestionable, that the "some not all" is positively required, even for the new system which has been proposed as supplementary to the old. At all events, the dealing with it must be left to those by whom the thorough development of the new system may be undertaken.

CHAPTER III.

CATEGORICAL INFERENCE, MEDIATE OR SYLLOGISTIC.

DIVISION I.—THE FORMAL DOCTRINE OF THE SYLLOGISM.

ARTICLE I.—*The Form of the Syllogism.*

The formal elements of the syllogism.

76. A simple Categorical Syllogism has three terms: the Major, the Minor, and the Middle.[1] Each of these occurs twice in the process. The minor and major terms are, respectively, the subject and the predicate of the consequent, and are often spoken of as the Extremes. The middle term is that which appears only in the antecedent. All the three names are significant. The middle term is introduced merely as a standard by which each of the other two may be measured: and, when an affirmative syllogism is reduced to its normal shape, this term is found to stand between the others, including the one, and being included in the other.[2] In a syllogism so reduced, the minor term

[1] This division of the chapter on the syllogism is designed to be an exposition of the received syllogistic scheme, embracing both the formal principles and all the special rules that have practical uses, and deviating as little as possible from the method followed in the standard books. Archbishop Whately's exposition, here as elsewhere, is admirable; and the details of processes are worked out with great exactness by Huyshe, *Treatise on Logic*, 1842. The doctrine is very instructively summed up from a higher point of view by Solly.

[2] The name "Argument," used commonly and conveniently to signify the process of inference as a whole, was currently applied

is seen to be that term which is included in the middle, the major term to be that which includes it.

The syllogism, when fully set forth, has three propositions. Two of these, which together constitute the antecedent of the inference, are called the Premises: and this name is applicable, not merely because they are made to stand before the consequent when the argument is set down for logical analysis, but also because they are the data and presuppositions of the process. The third proposition is the consequent.

The premise, whose two terms are the major and the middle, is called the Major Premise: the premise, whose two terms are the minor and the middle, is called the Minor Premise: the one proposition which is the consequent, and whose terms are the minor and the major, is called the Conclusion.[1]

The order of the propositions is a matter indifferent to the character of the argument. If we propose the consequent, in the shape of a problem or question, to be solved or

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by old logicians to the middle term. This meaning of the word has uses dialectical or rhetorical. The *discovery* of arguments, in proof of proposed conclusions, is resolvable into the discovery of middle terms.

[1] Other designative names have been given to the premises severally, but with varieties of application. The major has been called the Proposition, and, by some of the Germans, the Rule (a name bearing on the first figure). The minor has been called the Assumption, a name fitter perhaps for the major. It has more aptly been called the Subsumption (= position of minor under middle); and this name, like so many others of the science, has found its way into the nomenclature of business: the word lingers, though with almost total loss of meaning, in the forms of Scottish law-writs. Hamilton calls the major premise the Sumption, the minor the Subsumption.

answered through the antecedent, the conclusion, when ascertained, may stand first; and the premises will then follow as a reason, introduced by causal particles, as "because." But, in logical treatment of arguments, we assume the premises as given, and place them first; and we add the conclusion, introducing it by illative particles, as "therefore."

The order of the premises, again, has been fixed differently in different logical schools. The real course of the argument is best seen when the minor premise is put before the major. Some points of incidental illustration will, even now, become clearest when this order is adopted: and, in the last stage of our dealing with the categorical syllogism, it will force itself on us continually. But, the purpose, in the meantime, being to lay down and explain the received rules, the other order must be adopted. All those scholastic rules and schemes, which depend on arrangement of the propositions, suppose them to stand in this order: Major Premise; Minor Premise; Conclusion.[1]

The figure and mood of the syllogism.

77. In the Conclusion, as we have seen, the function of each of the terms is fixed. The minor is the subject, the major the predicate. The fact which fixes it is, the rejection of all propositional forms except A, I, E, and O. If the other possible forms were admitted, the function of the terms in the conclusion would be indifferent.

The function of the terms is not fixed in the Premises. The Middle Term, occurring once in each of these, may have

[1] Hamilton has summed up much information as to the order in which the premises have been arranged at different periods in the history of the science. (*Discussions*, p. 645; and note in Thomson's *Laws of Thought*, 1854; p. 224.)

its function varied in any of four several ways; and each of these variations may, in certain circumstances, be adopted without invalidating the argument. Accordingly, there are four admissible variations of the function of the middle term; and these yield the Four Syllogistic Figures. The *Figure* of a syllogism is its structure with reference to the function of the middle term. Figure, accordingly, is determined exclusively by the premises.

In the First Figure, the middle term is the subject of the major premise, and the predicate of the minor; in the Second Figure, it is the predicate of both premises; in the Third Figure, it is the subject of both premises; in the Fourth Figure, it is the predicate of the major premise, and the subject of the minor.[1]

The subject being always understood to stand before the predicate, the following table exhibits the position of the terms in each of the four figures. Here, and afterwards, M denotes the middle term, S the minor, P the major.

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[1] The fourth figure emerges, necessarily, when we look at the syllogism in this unanalytic way, asking only whether the middle term is subject or predicate. But the figure falls away, as being a variation of the first, if, dissecting the premises, we inquire which term is contained in which. So examined, the syllogism gives three figures only: the first (covering the fourth), in which the middle term is between the extremes; the second, in which it stands above both; the third, in which it stands under both. Aristotle, adopting this deeper analysis, and fixing no order of premises, recognised three figures only; and on this view we must in the end fall back. The source whence the schoolmen borrowed the fourth figure is doubtful. This figure has, by long tradition, been ascribed to Galen; but, after careful inspection of the fragmentary logical notices scattered through his medical writings, both Hamilton and Trendelenburg have failed to discover it.

	Figure I.	Figure II.	Figure III.	Figure IV.
Major Premise......	M P	P M	M P	P M
Minor Premise......	S M	S M	M S	M S
Conclusion..........	S P	S P	S P	S P

The structure of a syllogism, in reference to the quantity and quality of its propositions, is called its *Mood.* Those forms of predication only being taken into account which are denoted by A, E, I, and O, the moods arithmetically possible are sixty-four. For any one of the four forms might supposably be either major premise, minor premise, or conclusion; and each, appearing in any one proposition, might be accompanied in each of the other two propositions by any form of the four. But, as we shall immediately learn, a very large majority of the sixty-four moods would produce arguments totally invalid.

Article II.—*The Principle of the First Syllogistic Figure.*

The character of the first syllogistic figure.

78. By a large majority of logicians, the First Figure has been recognized as the normal form of the syllogism. In support of this opinion there is alleged the fact, that in this figure, and in it only, the middle term, the sign of the thought through which the other two terms are united in one judgment in the conclusion, occupies its just place in relation to the other two: it includes the minor term, and is itself included either in the major term or in its contradictory.

The terms are so related in the following syllogism of the first figure, in which, to exhibit the relation more clearly, the minor premise is placed before the major:—"All the S's are M's; all the M's are P's: therefore, all the S's are P's." "The minor is included in the middle; the middle is in-

cluded in the major: therefore, the minor is included in the major."

So is it, too, though the syllogism have a negative conclusion; as thus: "All the S's are M's; the M's are not P's: therefore, the S's are not P's." For the argument may be analysed in this way:—"The minor is included in the middle; the middle is included in the contradictory of the major: therefore, the minor is included in the contradictory of the major."

Let us examine, generally, the character of an argument thus framed. In the first place, it bears on the face of it a reference, more direct than that made by any other form, to the principle of non-contradiction. It is an unmistakeable passage from one identity or non-identity to another. It is a formula exemplifying a law: "things which are identical with the same thing are identical with each other." In our affirmative example, it was asserted that S is identical with a part of M, and the whole of M (including, of course, that part) with a part of P. Hence it was inferred, that S is identical with a part of P, or that the things called S are the same things which, with other things, are called P. To the negative example the same reasoning is applicable, with the substitution of Not-P for P.

In the next place, such an argument exhibits, in their natural order of sequence, the steps of a process of deduction. It is asserted that a given case is included in a class of cases, which are known to be governed by a law or principle assumed as already established. It is inferred that the given case is governed by that law or principle. "The given case S is included in the class of cases M; the whole class of cases M is governed by the law P (or Not-P): therefore, the case S is governed by the law P (or Not-P)."

79. Accordingly, there has been assigned, as the supreme and only original Law of the Syllogism, the maxim which, from one of its expressions, is called the *Dictum* (or *dicta*) *de omni et de nullo.* In all its shapes, it considers two of the terms as constituting an ordinated series. But it is usually framed so as to interpret the ordination from the side of extension; seldomer so as to interpret it from the side of comprehension. It must be examined in both aspects.

The dictum in its reference to the whole of extension.

(1.) The *dictum*, as it appears when the terms are read in extension, is most frequently enounced in such a shape as this: "Whatever is predicated (affirmatively or negatively) of a class, may be predicated in like manner (that is, affirmatively or negatively) of everything included in the class." A closer approximation to the scheme of the predicables is gained by this expression: "Whatever is predicated of a genus, is predicable also of a species included in the genus.[1]

In the major premise: Of a class or genus M—there is affirmed or denied—something denoted by P.
In the minor premise: The species S—is affirmed to be —included in the genus M.
∴ In the conclusion: Of the species S—there is affirmed or denied—that which is denoted by P.

The terms directly ordinated are the middle and the minor; the former is the genus, the latter an included species. The major term denotes an attribute, which is

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[1] Quidquid de omni valet, valet etiam de quibusdam et singulis (the "dictum de omni"): quidquid de nullo valet, valet nec de quibusdam nec de singulis (the "dictum de nullo").—Quidquid valet de genere, valet etiam de specie: quidquid repugnat generi, repugnat etiam speciei.

asserted to be possessed or not possessed by the genus, and which, therefore, through a double subalternation, is inferred to be possessed or not possessed by the species. "All the M's have the attribute P (and, consequently, some M's have it); but the S's are some M's: therefore the S's have the attribute P."

Again, in the dictum, as thus read, the major term is not expressly embraced in the ordination. It does not directly require to be so. In both of its appearances it is a predicate, interpretable as the name of an attribute; and therefore it does not necessarily receive a place in a series constituted by terms which are regarded as the names of classes, containing objects or substances. But its place, as the most extensive term in the ordination, is unavoidably implied: it is impossible to read, analytically, any syllogism exemplifying the dictum, without bringing this relation to light. The three propositions of a syllogism purely affirmative, are assertions of three successive and widening steps of inclusion: the propositions of a syllogism which introduces negatives are readily and correctly interpretable in the same way, if only we substitute for the major term its contradictory.

The completed ordination of the terms in extension, from narrowest to widest, is this: "S, M, P (or Not-P)." This gradation is explicated, step by step, when the syllogistic propositions are arranged thus: minor premise, major premise, conclusion. "All (or some) S's are in M; all M's are in P (or in Not-P): therefore all (or some) S's are in P (or in Not-P)."[1]

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[1] This resolution of negative syllogisms into affirmatives is, evidently, through contraposition of the major premise and the conclusion. It is possible in the first figure, because in it the major term

Otherwise, indeed, syllogisms in which negatives are introduced might, easily, be traced back to a pre-ordination without displacement of the negations. The middle term and the major would, in this view, be taken as co-ordinates, which, by the law of the concept, must be denied of each other. The minor would then denote a species: the middle and major would denote two genera proximate to it, and mutually co-ordinate and exclusive. The ascending ordination would be: "S, M + P."[1]

80. (2.) When the terms are read in comprehension, the dictum takes several forms, of which this is the most common: "The mark of a mark is a mark of a thing."[2] That is, an attribute of a second attribute is an attribute of any object or substance possessing the second attribute. The

The dictum in its reference to the whole of comprehension.

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is everywhere a predicate. Since, therefore, it is always competent thus to substitute, for a proposition of exclusion, its equipollent proposition of inclusion, the *dictum de nullo* might be dispensed with. But, though the contraposition is often convenient, and though, especially, it gives the clearest view over the ordination of the terms; yet it is not a safe operation where the validity of a chain of syllogisms is under scrutiny. If we were to contrapose a negative conclusion, we might, when it next emerges as a premise, require to re-contrapose.

[1] For a characteristically acute statement of difficulties, which the last two or three paragraphs, as well as similar resolutions elsewhere, are designed to meet, see Trendelenburg, *Logische Untersuchungen*, ii., 238, &c. His objections are two: first, that when the dictum is considered in extension, the major term falls out of the scale of subordination; secondly, that when one of the premises is negative, the subordination breaks down altogether.

[2] Nota notæ est etiam nota rei: (repugnans notæ repugnat rei).

first attribute is the major term; the second attribute is the middle term; that which is regarded as a substance is the minor term. The negative expression of the maxim is needless, and tends to perplex: it is sufficient, as before, to substitute, when negation is introduced, the contradictory of the major term for that term itself. "If P (or Not-P) is a mark or attribute of M, and if M is a mark or attribute of the object or objects S, then P (or Not-P) is a mark or attribute of S."

The meaning of the rule in this shape is plain; and its truth, as a simple application of the law of identity, is self-evident. The use of it in testing syllogistic examples is troublesome; because it throws us on those abstract phrases, which, though distinctively significative of the relation of comprehension, are unusual and unmanageable.

We are guided towards concrete phrases, by an expression of the dictum supplied to the schoolmen by Aristotle himself,—an expression which, while it is more readily applicable to comprehension than to extension, does not expressly allege either. It is the widest, and perhaps the most apt, of all the shapes in which the dictum can be couched. "That which is predicated of the predicate, may be predicated of the subject." That which (in the major premise) is predicated of the predicate (of the minor premise), may (in the conclusion) be predicated of the subject (of the minor premise).[1]

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[1] Prædicatum prædicati est etiam prædicatum subjecti. "Ὅσα κατὰ τοῦ κατηγορουμένου λέγεται, πάντα καὶ κατὰ τοῦ ὑποκειμένου ῥηθήσεται. (*Categ.*, cap. 5). In toto esse vel de omni prædicari dicitur, quoties non potest inveniri aliquid subjecti, ad quod illud quod prædicatur dici non possit. In toto vero non esse vel de nullo

In the major premise: Of M—there is affirmed—the mark or attribute P (or Not-P).
In the minor premise: Of S—there is affirmed—the mark or attribute M.
∴ In the conclusion: Of S—there is affirmed—the mark or attribute P (or Not-P).

In the comprehensive form of the dictum, the terms directly ordinated are the middle and the major. The major is in the comprehension of the middle, being a mark of it: the major is the less comprehensive, the middle the more comprehensive, of the two. The minor term denotes a substance or substances, an object or group of objects, which is asserted to possess the attribute denoted by the middle; and it is therefore inferred to possess the attribute denoted by the major, which is a part of the attribute denoted by the middle. "The attribute M comprehends the attribute P (or Not-P); the S's (all or some) possess the comprehending attribute M: therefore the S's (all or some) possess the comprehended attribute P (or Not-P)."

When the dictum was analysed extensively, the major term, being used only as an attributive name, did not require to be expressly referred to an ordination constituted by terms which were considered as names of substances. Now, when the dictum is analysed comprehensively, the minor term, which in both of its appearances is only a name denoting substance, does not require to be expressly referred to an ordination containing terms which are considered as names

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prædicari dicitur, quoties nihil subjecti poterit inveniri, ad quod illud quod prædicatur dici possit." (Boethius, *De Syllogismo Categorico,* Opera, p. 591).

of attributes. But it may be so referred: and the presupposed series is not complete until the minor term is placed as one extreme of it.[1] The minor term, which was found to be the least extensive of the series, is necessarily the most comprehensive. The completed ordination of the terms in comprehension, from narrowest to widest, is this: "P (or Not-P), M, S." This gradation is, of course, explicated, when the propositions are arranged in the order opposite to that which exhibited the widening scale of extension: conclusion, major premise, minor premise. The retention of the negative predications might here, as in the other view, be justified through the assumption of co-ordination, and consequent exclusion, between the middle term and the major.

The special laws of the first figure inferred from the dictum.

81. The *dictum de omni et nullo*, in all its forms, is plainly nothing more than a corollary, or pair of corollaries, from the principle of predication through common terms. It is required only to accept two, at most, of the special laws deduced from that principle; and to explain their applicability to cases in which the antecedent is not one proposition but two. *The Law of Affirmation through Subalternation*, expressed so as to cover more degrees than two, guides us through all syllogisms purely affirmative: while, as it has been shown, the negations of a syllogism may be displaced, and this one law be applied to all. If given negations are retained, we have only to accept, along with the law of affirmation through subalternation, *the Law of Negation through Co-ordination.*

[1] Consult, again, for the questions raised here and in the last section, Trendelenburg, ii., 239. He there animadverts on the "Nota notæ," (the form of the dictum adopted by Kant), which he rightly recognises as a reading of the dictum in comprehension.

The first figure is subject to three Special Laws. These, though they are proveable, and will be proved, otherwise, are deducible, readily and instructively, from the dictum. The extensive form of the dictum is most conveniently available for the purpose.

(1.) The Minor premise must be Affirmative. In that premise, the minor term is subject, the middle is predicate. The rule therefore says, simply, that the minor term must be included in the middle, not excluded from it; that it must be describable (if it be a common term), as one of those species which the middle term, a genus, is supposed to contain. If it were not so included, so describable, nothing would be asserted to be "contained in the class," no species to be contained in the genus. Whether the major term were affirmable or deniable of the middle, there would be no data for determining whether it were affirmable or deniable of the minor. In a word, there would be wanting the lowest step of the ordination in extension, the step out of which the others come.

The middle term signifies a class; the major signifies an attribute belonging to the class, or wanting in it. The minor signifies something which is placed in that class; and, being so placed, it must possess the class-attribute, or want an attribute not found in the class. The middle term is introduced to be a link in thought between the minor and the major: it is clasped to both. But, if the minor is not in the middle, the hold on that side has snapped.

(2.) The Major premise must be Universal. In that premise, the middle term is subject, the major is predicate. Consequently, the rule alleges, that the whole class designated by the middle term must possess the attribute denoted by the major; or, in other words, that in this premise

the middle term must be distributed. The reason stares us in the face. For the minor premise asserts, not that the minor term constitutes a class M, but only that it is in that class, or makes some part or other of it. If, then, it were only alleged, in the major premise, that some part or other of the class M possesses the attribute P (or Not-P), this might or might not be the same part which has been affirmed of S. The identity of S with a part of M possessing P, or its contradictory, cannot be made peremptory at any cheaper rate, than the assertion that the whole of M possesses P, or its contradictory.

But, it is to be observed, this necessity, for distribution of the middle term in the major premise, arises out of two features in the minor premise. *First*, That premise has an undistributed predicate (M): it affirms only that S is some part of the class M. *Secondly*, It affirms only that S is some undetermined part of M. Suppose that the part of M were there definitely identified; and that a part of M similarly identified were to become subject of the major premise. It might then be determinable, and signifiable through the form of expression, whether the part first named and that named second were one and the same, or two several parts. In the former case, the chain of identity would be unbroken: in the latter, it would be broken, and further progress from contained term to term containing would be impossible.

(3.) The Quality of the major premise determines the quality of the conclusion; the Quantity of the minor premise limits the quantity of the conclusion. This law, in both of its parts, is almost self-evident to every one who understands the character of the syllogism.

In the major premise it is asserted, either that the middle term has, or that it has not, the attribute denoted by the

major term. If it has the attribute, the minor term also must have it, being a part of the middle: if the middle wants it, so must the minor. The minor premise, again, affirms, that either the whole class denotable by the minor term, or a part only of that class, is in the middle. If it has affirmed of the whole class, the predication in the conclusion may be made of the whole class (though, of course, we might hence subalternate to a part): if it has affirmed of a part only, it is of a part only that we can predicate in the conclusion.

Article III.—*Laws, Universal and Special, of the Syllogistic Figures.*

82. The *dictum* may be brought to bear on any syllogism whatever. For every syllogism of the second, third, or fourth figure, admits of being transformed into a syllogism of the first. But it is desired by those who hold the pre-eminence of the first figure, and insisted on by some of those who deny it, that we be enabled to test every syllogism as given, without questioning as to its figure, or making any change on its structure. **The two syllogistic canons.**

Such a test is supplied by two rules, the first justifying conclusions which are affirmative, the second those that are negative. They have been called the *Canons of the Syllogism.*

They are these: "Two terms which agree with the same third term, agree with each other. Two terms, whereof the one agrees and the other disagrees with the same third term, disagree with each other."[1]

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[1] These expressions of the canons are, in substance, Aldrich's and

Two terms are said by many logicians to "agree," or to be "congruent" or "consistent," when they may be the terms of an affirmative proposition. Two terms are said to "disagree," or to be "incongruous" or "inconsistent," when they can only be the terms of a negative proposition. This phraseology being adopted in the canons, agreement of terms in a given proposition is affirmation, disagreement is negation. The two terms first mentioned are the Minor and Major, the third term is the Middle.

The canons, accordingly, are interpretable by direct reference to the character of the copula in each of the three propositions of the syllogism. They might thus take the following shapes:—

First: "If there are given, as premises, two propositions,

Whately's. They are thus given by Burgersdyk, as bearing on syllogisms having a singular as middle term,—the "expository syllogisms" of the schools: "Quæcunque uni tertio singulari conveniunt, ea quoque inter se conveniunt. Quorum unum alicui tertio singulari convenit, alterum non convenit, ea quoque inter se non conveniunt." Smiglecius carries them up directly to Identity and Difference: "Quæ sunt eadem uni tertio, sunt eadem inter se. Quando est idem alicui, cui aliud non est idem, ipsa quoque non erunt idem inter se." (*Disputatio* xiii., *Quæstio* 14.) Keckermann unites the two, calling the complex law "The Principle of Proportion," that is, the principle in virtue of which the middle term becomes a measure of the other two: "Quæcunque in uni tertio conveniunt, inter se conveniunt: quæ vero in uni tertio dissentiunt, inter se dissentiunt." (*Systematis Logici* lib. iii., cap. 5.) This form requires a gloss: the agreement or difference "in" the middle term, is agreement or difference in the quality of the premises. The reference of the syllogism to the canons, as universally superseding the dictum, was a step not taken till the latest times of scholasticism; and it has very frequently been protested against.

in which there is asserted, between M as one term, and S and P successively as the other term, a relation expressed by affirmation, there is implied, between S and P, a relation which must be expressed by affirmation in the conclusion."

Secondly: "If there are given, as premises, two propositions, in which, between M as one term, and S and P successively as the other term, there are asserted relations, which are expressed in either of the premises by affirmation and in the other by negation, there is implied, between S and P, a relation which must be expressed by negation in the conclusion."

So interpreted, the canons are evidently direct applications of the laws of identity and difference; for affirmation is the expression of identity, negation of difference.

The identities and differences are explicable in this way. 1. Two things, or groups of things, which are identical with a third thing, or group of things, must be identical with each other. If it be assumed that both S and P are identical with M, S and P must be identical with each other; or the names S and P are but two names for the same thing, or group of things. 2. If, of two things, or groups of things, the one is identical, the other non-identical, with a third thing or group of things, the two must be non-identical with each other. If it be assumed either that S is identical, and P non-identical, with M; or that S is non-identical, and P identical, with M: S and P must be non-identical with each other; or the names S and P are names for two different things, or groups of things.

If we accept, and steadily keep hold of, the quantitative signs, "all, any, some," as integral parts of the terms, the applicability of the canons, as expressions of the two primary laws of predication, is thorough-going. It is plain, too, that

the canons, in all their expressions, leave the function of each of the terms, as subject or predicate, completely open for all the three propositions. The only limitation is, the prohibition of all forms except A, E, I, and O.

The weakness of the canons lies, first, and positively, in their leaving the quantitative signs unaccounted for. The reasons which make these possible and necessary, do not emerge till we fall back on the dictum, and on those laws of common terms out of which it has grown. But it may even be questioned, whether the canons do not necessarily, —whenever the use of them requires interpretation of the quantitative signs,—presuppose the dictum, or the law of logical totality, from which the dictum itself springs.[1]

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[1] These two paragraphs may call for some illustration.

(1.) Given this syllogism in AII: "All M's are P's; some S's are M's: therefore some S's are P's." The data here are not the classes S, M, and P: the identity or non-identity of these classes as wholes is not put in question. The quantity of predicates being brought out, it appears that our terms are these: "some S's;" "all M's," ("some M's," however, being implied, and requiring to be explicated); "some P's." The assertions of identity are these: (1.) The objects called "all M's," are the same objects which are also called "some P's;" (2.) The objects called "some S's," are the same objects which are also called "some M's:" therefore (3.), the objects called "some S's," are the same objects which are also called "some P's."

(2.) Even when the quantitative signs are thus incorporated, there is still required, as the example shows, one interpretation of them. We cannot gain our inference, without assuming that "all" implies "some." If this assumption is refused us, we cannot show that the premises assert the identity of S and P with the same third term. But how do we come by the assumption? Can we do so without the idea of classification? Or can classification supply it, unless through the dictum or its principle?

83. The canons, then, enable us to consider every syllogism, whatever be its figure, as constituted by three assertions of identity or difference between the three terms, taken two and two. But, in order to determine positively what the objects are, which are asserted to be identical or different, we have to hold the quantitative sign of each term as being a part of it; and, in order to test decisively the question of identity or difference, we have to assume that "all" includes "some," or that a whole includes its part.

The six universal rules deducible from the canons.

On this footing there are deducible from the canons six

If this doubt is well-founded, the canons are even weaker than the text has positively alleged them to be. They are sufficient, without the dictum, to justify syllogisms having a singular middle term. They cannot justify syllogisms whose middle is a common term, unless through a presupposition, of which the dictum, or the higher law of conceptive totality, is an element.

However, the assertion may be held to rest on the still wider truth, that a whole contains its part.

(3.) In spite of theoretical difficulties, the convenience of the canons is great. And, if the quantitative signs are taken as integral parts of the terms, and if, also, interpretation of these is admitted to the extent of the implication of "some" in "all," they do, as alleged in the text, carry us through syllogisms of any form. When, again, the conclusion is an E or an I, it is a matter of indifference to the application of the canons, which of the terms S and P is subject of the conclusion, and which predicate: the terms of E and I are reciprocal; and simple conversion yields a new E or I. But conclusions in A and O tie us down to S as subject, and P as predicate. A is convertible only into I^2, a form not received: O is not directly convertible at all. The fact that conversion would change the character of the predication in reference to the wholes of the concept, is not forced on our consideration when we test a syllogism by the canons.

maxims, which are describable as Universal Rules of the Syllogism. Violations of these are, for mediate inference, the only Fallacies, or faults making an argument inconclusive, which logic can detect without extraneous aid.

(1.) A Syllogism has Three Terms. It is invalid if it has either fewer or more. A proof of this rule would be merely a repeated description of the nature of the syllogism; and, if the rule is accepted with presupposition of its reasons, the rules of quantity may be regarded as corollaries from it. The violation of any of them is resolvable into the introduction of a fourth term.

Fewer terms than three no syllogism is in any danger of having. But, seeming to have three only, it may really have more than three, through the ambiguity of words. This rule, indeed, is designed chiefly as a preliminary caution, against the fallacies which may insinuate themselves into reasoning by means of the fact, that there is no term, however simple, which the poverty of speech does not make to be capable of bearing more than one signification. If any name furnishes a term susceptible of two meanings, and if it bears one of these on its first appearance in a syllogism, and the other in its second, it has really furnished not one term, but two; a term denoting one set of objects, another term denoting a different set. The syllogism, therefore, has four terms, not three; and, at some point or other, the chain of comparison is broken. The middle term is, unquestionably, in greater danger of having two meanings, than either of the other terms is: and the intrusion of a fourth term in disguise is often described as the Fallacy of Ambiguous Middle. But the other terms likewise require to be watched.

(2.) The Middle term must be Distributed in one of the

premises. The violation of this rule is the Fallacy of Undistributed Middle.

The rule is a consequence, following from the indefinite character of the part signified by the logical "some." M is introduced into the premises, in order that S and P may severally be compared with it. If it is twice undistributed, it may, for all we know, be double; and, if it is double, the syllogism has four terms. If S were compared with "some part or other" of M, and, again, P with "some part or other" of M, it would be left absolutely uncertain whether they have been compared with the same part, or with two parts that are different. No ground would have been laid down for determining their mutual difference or identity. But, if one of them is compared with the whole of M, and pronounced to be either the same or not the same with that whole, it is enough that the other should be compared with a part of the whole of M: since the first term, having been compared with the whole, must have been compared with this part. Accordingly, one distribution of the middle term is sufficient.[1]

(3.) Neither the Minor term nor the Major must be Distributed in the Conclusion, if it was Undistributed in its Premise. Violation of this rule is an Illicit Process, and is, therefore, of two kinds: Illicit Process of the Minor, Illicit Process of the Major.

When we remember that the quantitative sign is a part

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[1] In the first and second figures, the middle never *is* twice distributed. When it is so in other figures, the conclusion obtainable is exactly the same as it would have been if the middle had been distributed only once. The moods, in which the double distribution occurs, have premises needlessly wide for the conclusions.

of every term given in the process, we shall perceive at once that a breach of the rule would amount to the introduction of a fourth term. There is given in the premise a relation between M and "some S's" or "some P's." This gives us no right to infer, in the conclusion, anything in regard to "all S's," or "all P's."[1]

(4.) If both Premises are Affirmative, the Conclusion must be Affirmative.

This is evident, both from the words of the canons, and from their principle. The case is that which the first canon covers. Assertion of the identity of M both with S and with P, cannot, without self-contradiction, yield assertion of the non-identity of S and P with each other.

(5.) If either of the Premises is Negative, the Conclusion must be Negative.

The case is covered by the second canon. If S (or P) is M, and if P (or S) is not M, S and P must be, not one thing, but two different things.

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[1] The two kinds of Illicit Process are not fallacies lyingequally deep. In committing an illicit process of the minor, we have only inferred a universal, A or E, in a case where, if there be no other fallacy, we might have inferred a particular, I or O. The error is easily corrigible. But an illicit process of the major is, when it is the one fault, a fault that is incurable. It is forced on us by our having premises from which no inference at all could be drawn. It is possible only when the conclusion is negative, E or O; these conclusions only having a distributed major. In the premise, then, the major term, being by hypothesis undistributed, must have been either the subject of a particular, I or O, or the predicate of an affirmative, A or I. It will immediately appear, from the special rules of the figures, that, in such circumstances, inference is always impossible.

(6.) From Premises, both of which are Negative, no Conclusion can be inferred.

The case, not being covered by either of the canons, is thus virtually excluded; and it is self-evident that the exclusion is right. The assertion that both S and P are non-identical with M, leaves it utterly undetermined whether they are or are not identical with each other.

A violation of the first rule is a fallacy which throws the reasoning out of the very form of a syllogism. Violations of the second and third rules are fallacies of Quantity. Violations of the fourth, fifth, and sixth rules are fallacies of Quality. When the error does not flow from the most abundant of all sources, the ambiguity of terms, the fallacies most likely to occur are those of quantity; namely, undistributed middle and illicit process.[1]

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[1] There have been laid down several other rules, all of which are truly derivative, either from the six above given, or from other obvious laws.

For example: When a universal Conclusion may be inferred, there may be inferred also a Particular. This is nothing more than an assertion of the possibility of immediate inference, from the truth of the subalternant to the truth of the subalternate. But the point is worth observing; since it has a bearing on the scheme of the syllogistic moods.

Two of the scholastic rules deserve the subordinate place assigned to them by Whately. The violation of either of them warns us, that there must have been committed one or another of the fallacies of quantity. They may be numbered as supplementary to the six of the text.

(7.) When one of the premises is Particular, the conclusion must be Particular. The transgression of this rule is a symptom of illicit process of the minor.

This, and the fifth rule of the text, are combined by many of the

There are annexed a few examples, in which the Six Rules are palpably violated.[1]

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old logicians into the one rule: That the conclusion, both in quality and in quantity, follows the Worse, or weaker, premise; "Conclusio sequitur partem deteriorem." Negation is "worse" than affirmation, particularity than universality.

(8.) From premises, both of which are Particular, no Conclusion follows. The transgression of this rule is a symptom, either of undistributed middle, or of illicit process of the major, the two fallacies which arise out of bad premises.

Premises in OO, as being both negative, are condemned otherwise. Premises in II plainly give undistributed middle. Premises in IO (the major premise first), require O in the conclusion, and give illicit process of the major; while in Figure III. there will also be undistributed middle. Premises in OI give undistributed middle, when that term is subject of the major premise (that is, in Figures I. and III.): they give illicit process of the major, when that term is subject of the major premise (that is, in Figures II. and IV.).

[1] *Breaches of the Six Syllogistic Rules.*

Rule 1. Nettles are stinging plants; and fig-trees are nettles: therefore it must be true that fig-trees sting. "All M's—are—some P's; all S's—are—some M's: ∴ all S's—are—some P's." (Ambiguous middle, making four terms).—The fig belongs to a tribe of plants, the typical name of which (Lindley's "Urticales") is taken from the common nettle.

Rule 2. Since many plants are beautiful, and many plants are rare; may we not infer, that among the things that are rare, there are some that are beautiful? "Some M's—are—some P's; some M's—are—some S's: ∴ some S's—are—some P's." (Undistributed middle).

Rule 3. (1.) Interest attaches to all plants, and rarity may be asserted of many plants; is it therefore true that every thing rare

84. When the sixty-four possible Moods of the syllogism are tested by the six rules, it is discovered that

Determination of the eleven valid moods.

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is also interesting? "All M's—are—some P's; some M's—are—some S's: ∴ all S's—are—some P's." (Illicit process of minor).

(2.) We infer that there is no beauty in flower-beds; because grassy surfaces are beautiful, and flower-beds are not grassy surfaces. "All M's—are—some P's; any S's—are not—any M's: ∴ any S's—are not—any P's." (Illicit process of major).

Rule 4. Assuming that every exotic plant is interesting, but that not a few such plants are useless; should we be in any danger of inferring that few things which are useless are also interesting? "All M's—are—some P's; some M's—are—some S's: ∴ some S's—are not—any P's." (Both premises affirmative; conclusion negative).

Rule 5. If all imperfect plants want true flowers, and if true flowers grow on all trees; we are surely not tempted to infer that trees are imperfect plants. "Any P's—are not—any M's; all S's—are—some M's: ∴ all S's—are—Some P's." (One premise negative; conclusion affirmative).

Rule 6. If we wish to determine whether the potatoe-plant is or is not a nightshade (*Solanum*), we shall hardly suppose ourselves to have sufficient data, though we know that neither any potato, nor any nightshade, produces fruit which can safely be eaten. "Any P's—are not—any M's; any S's—are not—any M's." (Both premises negative; no conclusion possible).

In a preceding article of the Encyclopædia [FALLACY], the principal sources of inconclusiveness in argumentation were described in outline. The topic is hardly anywhere treated so thoroughly, and nowhere so practically, as in Whately's dissertation, "Of Fallacies."

Fallacy may have place, either in the *form* of an argument, or in its *matter*. If it is formal, the conclusion does not follow from the premises; and the logical rules expose the flaw. If it is material, the conclusion does follow from the premises; and logical rules

fifty-three of them do necessarily, whatever the figure may be, yield bad conclusions. None of them can violate the first rule: but all of them violate either rules of quality,

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have no bearing: but, here, the fault may be, either in the premises, or in the conclusion. Accordingly, all the kinds of fallacy may be distributed in the following way. Either they are *Fallacies of Inference,* which are formal, but may be either *patent* or *latent;* or they are *Fallacies of Assumption* or *of Exposition,* both of which are material. Not infrequently, an argument combines fallacies of more kinds than one.

(1.) A fallacy of inference is patent, when the logical rules detect it without interpretation of terms. It is latent, when interpretation of terms must precede logical analysis. The latency of formal fallacies arises always from the *ambiguity of words;* and the treatises on fallacies, which are so full in many logical works, are chiefly occupied in explaining the causes and kinds of ambiguity. (2.) A fallacy of assumption consists in arguing from one or more premises which are not true or not admitted. It is most frequently latent; the faulty premise being virtually, but not obviously, either identical with the conclusion or dependent on it. This latency is the case of "petitio principii" or "quæsiti," the begging of the question; and the name "arguing in a circle" should mean a repetition of this error, by inferring back to something which was really a preceding premise. (3.) A fallacy of exposition is a mis-statement of the question which is at issue. Its scholastic name is "ignoratio elenchi," the ignoring or shunning of the conclusion which would contradict the position of the adversary: the argument has as its result an "irrelevant conclusion," a conclusion different from that which ought to have been proved. This fallacy is signified by the common expression of "shifting the ground." A union of it with ambiguity of terms is perhaps the most frequent of all flaws in reasoning. To it are referable instances in which any principle, rightly pleadable in some cases, is applied to a case which it does not cover. This is exemplified by the "argumentum ad verecundiam," the appeal to authority; and by the "argumentum ad hominem," the

or rules of quantity, or rules of both kinds. Accordingly, no more than *eleven* of the sixty-four can ever be valid.[1]

First, There are thirty-two moods which appear, on the face of them, to involve fallacies of quality. Our fourth rule is transgressed by AAE, AAO, AIO, IAO; our fifth by AEA, AEI, AOA, AOI, EAA, EAI, EIA, EII, IEA, IEI, OAA, OAI; our sixth by EEA, EEE, EEI, EEO, EOA, EOE, EOI, EOO, OEA, OEE, OEI, OEO, OOA, OOE, OOI, OOO.

Secondly, There are twenty-one moods which, when the distribution of the terms is examined, are found to involve fallacies of quantity.

(1.) The supplementary rule, numbered as our seventh, gives notice that the third rule is violated by AIA, AIE, AOE, EIE, IAA, IAE, IEE, OAE.

(2.) The supplementary rule, numbered as our eighth, gives notice that either the second rule or the third must be violated by IIA, IIE, III, IIO, IOA, IOE, IOI, IOO, OIA, OIE, OII, OIO.

(3.) The mood IEO necessarily violates the third rule through illicit process of the major. The major term is distributed in the conclusion; while it cannot, whether as subject or as predicate, have been distributed in its premise.

Not a few of the moods thus excluded, violate evidently more rules than one.

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attempt to show that an opponent's position is taxable with inconsistency: and in a similar predicament are illegitimate endeavours to influence judgment through emotion.

[1] Henceforth, in this division, the common order of the premises, as being that which is taken for granted in the received doctrine of mood and figure, must be steadily adhered to. The major premise precedes the minor.

The other moods, which only can ever yield a valid inference, are these eleven: four moods having affirmative conclusions, AAA, AAI, AII, IAI; seven moods having negative conclusions, AEE, AEO, AOO, EAE, EAO, EIO, OAO.[1]

Determination of the twenty-four valid moods in figure.

85. It must now be considered how the relation of Mood is affected by the relation of Figure.

We see that there are no more than eleven combinations of propositions, which can, in any figure, stand the test of the syllogistic rules. Further, the functions imposed on the terms, as subject or predicate, in the premises of the Four Figures, vary far enough to make us expect, before minute inspection, that a mood may be valid in one figure yet invalid in others. And the fact is so.

The moods are not applicable to use, unless through the adoption of one or another of those modes of constructing premises which constitute the figures. Accordingly, we may henceforth understand, by a mood, a Mood in a given

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[1] The moods might be scrutinized, also, in another and quicker way. The exclusion of the bad moods is gained, by far most readily, through this question: How many pairs of premises are there, which, as being neither both negative nor both particular, may yield some conclusion or other? There are nine such pairs; but one of them, IE, breaks down when closely handled. Therefore the valid pairs of premises are only eight: AA, AE, and EA, both universals; AI, IA, AO, EI, OA, of which the one is universal, the other particular. Each of the three pairs of universals admits two conclusions, a universal and a particular: each of the others is tied down to one particular conclusion. The conclusions being supplied, we have the same eleven moods which had been discovered through the more cumbrous process.

Figure. Of the eleven valid moods, each of the figures admits six: whence, of Moods in Figure, there are in all Twenty-Four. Five of these, however, noted in the following list by italic letters, are unused, as giving particular conclusions where the premises allow the conclusions to be universal. These particular conclusions, indeed, may always be held to have been reached through subaltern inference from the universals.

Figure I. has these moods;	AAA, *AAI*, EAE, *EAO*, AII, EIO.
Figure II.	EAE, *EAO*, AEE, *AEO*, EIO, AOO.
Figure III.	AAI, IAI, AII, EAO, EIO, OAO.
Figure IV.	AAI, AEE, *AEO*, IAI, EAO, EIO.[1]

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[1] This table allows a comparison of the figures, in reference both to the structure of their syllogisms, and to the forms of propositions which are attainable as conclusions through them. Of many considerations w ich suggest themselves, the following are the most important:—

I. Of the eleven moods, there are only two which distribute their terms widely enough to sustain, without lapsing into fallacies of quantity, all the changes of structure which the variation of figure causes in premises. These, as we should expect, are negative moods; they are EAO and EIO. But, though these moods are good in all the four figures, the former is useless in two of them. Still, the fact stands, that, in all figures, EA, and even EI, are good premises.

II. The figures differ widely, in respect of the range of their conclusions.

(1.) The pre-eminent power of the First Figure is shown in two features. It is the only figure in which we can prove a universal affirmative; and it is the only figure in which we can prove conclusions having all the four forms, A, E, I, and O.

(2.) The Second Figure allows no conclusions but negatives.

(3.) The Third Figure allows no conclusions but particulars.

The special rules of the four figures.

86. These twenty-four valid Moods in Figure are easily ascertained, through application of the six universal rules of the syllogism to those modifications of the functions of the terms in premises, which severally constitute the four syllogistic figures. The limitations which the six rules impose on the character, both qualitative and quantitative, of the propositions admissible to constitute a syllogism in each of the figures, might be gathered from the table, just set down, of the moods in figure. But some uses are served by the generalizing of those limitations into Special Rules of the Figures, and by a brief deduction of these from the six universal rules.[1]

(4.) While the Fourth Figure allows all forms of conclusion except A, its weakness is shown by this fact, among others: that in it we can prove an I only, from the same premises which, by being merely transposed, would fall into the first figure, and give a conclusion in A.

III. The several pairs of premises are very unequally applicable to the proof of conclusions belonging to the four several kinds. The data sufficient for establishing propositions Particular, or Negative, are, in a humbling degree, more various than those that are required for establishing Universals, or Affirmatives. The five subalternated conclusions being thrown out of account, the reckoning for the other nineteen stands as follows:—

(1.) The moods proving universal conclusions are five; those proving particulars are fourteen.

(2.) The moods proving affirmatives are seven; those proving negatives are twelve.

(3.) A is proveable by one mood; E by four moods; I by six; and O by eight.

[1] Among the neatest demonstrations of the Special Rules of Figure from the six rules, are those of Huyshe. The third rule of the first figure is not given in the English books.

Figure I.

The three special rules of the First Figure have already been proved through the dictum. They are proveable, also, through the canons and the deduced universal rules; not with so close a reference to principles, but much more briefly than in the other way.

(1.) The Minor premise must be Affirmative. Suppose it negative: the major premise must then, by the sixth rule, be affirmative; and the major term, being its predicate, will be undistributed. But, by the fifth rule, the conclusion must be negative, and will accordingly have its predicate, the major term, distributed. Therefore, a breach of this special rule would cause the incurable fallacy of an illicit process of the major.

(2.) The Major premise must be Universal. Let it be particular; in other words, let its subject, the middle term, be undistributed. By the special rule just proved, the minor premise must be affirmative, and will not distribute its predicate, the middle term. Therefore, a breach of this rule produces the incurable fallacy of undistributed middle.

(3.) The Quality of the major premise determines the quality of the conclusion; the Quantity of the minor premise limits the quantity of the conclusion. (1.) If the major premise were negative, and the conclusion affirmative, the fifth rule would be violated: if the major premise were affirmative, and the conclusion negative, then (besides the qualitative fallacy caused by the two affirmative premises) the major, being predicate of both propositions, would suffer illicit process. (2.) The minor term, being subject both of the minor premise and of the con-

clusion, would suffer illicit process, if it were undistributed in the former and distributed in the latter. If the minor premise is universal, the conclusion may be so: and subalternation would thence yield a particular.

Figure II.

The special rules of the Second Figure are three.

(1.) One of the Premises must be Negative. The middle term, being the predicate of both premises, would be twice undistributed if both were affirmative.

(2.) The Conclusion must be Negative. One of the premises being negative, the conclusion also must be so, by the fifth rule.

(3.) The Major premise must be Universal. It must be so on pain of illicit process of the major term. The conclusion, being negative, distributes that term: consequently, it must be distributed in its premise, of which it is the subject.

Figure III.

The special rules of the Third Figure are two.

(1.) The Minor premise must be Affirmative. If it were negative, the major premise must, by the sixth rule, be affirmative, and would not distribute its predicate, the major term. But, by the fifth rule, the conclusion must be negative, by reason of the negative premise: consequently, the major term would here be distributed; in other words, there would be committed an illicit process of the major.

(2.) The Conclusion must be Particular. By the rule just proved, the minor premise is affirmative; and the

minor term, being its predicate, is undistributed: consequently, if that term were distributed in the conclusion, there would be an illicit process of the minor.

Figure IV.

The awkwardness of the Fourth Figure is never more striking, than when the attempt is made to lay down special rules for its construction. The three following are the most comprehensive of those that have been applied to it:—

(1.) If the Major premise is Affirmative, the Minor premise must be Universal. The middle term, being the predicate of the major premise, is undistributed when that premise is affirmative. If, then, the minor premise were particular, the middle, being subject, would be again undistributed. The rule must be obeyed, for the avoidance of undistributed middle.

(2.) If one premise is Negative, the Major premise must be Universal. If the major premise were particular, the major term, being its subject, would be undistributed. But, by the hypothesis and the fifth rule, the conclusion must be negative, and the major term must there be distributed. The penalty for breach of the rule is illicit process of the major.

(3.) If the Minor premise is Affirmative, the Conclusion must be Particular. The minor term, being predicate of its premise, is, by the hypothesis, undistributed: consequently, it cannot be distributed in the conclusion. The rule is fenced by illicit process of the minor. It is this limitation that makes premises in AA so much less valuable in this figure, than they are when transposed into the

first, where the term which was subject of the conclusion becomes predicate, and is, therefore, left undistributed without harm.[1]

The reduction of syllogisms.

87. Those logicians who maintain the pre-eminent validity of the first figure, describe syllogisms in it as Perfect and Direct; syllogisms in the other three figures as Imperfect and Indirect.[2] All syllogisms of the indirect figures admit of being transformed into syllogisms of the first figure. The process of transformation is called Reduction.

The method which must chiefly be available is evidently the conversion of one or both of the premises. In reducing from the second figure, where the middle term has already its right place in the minor premise, the conversion of the major premise would seem to be sufficient. For the third figure we should require conversion of the minor premise; for the fourth figure, conversion of both.

But, conversion having been performed, we should sometimes have gained premises which, though good in

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[1] These are the rules of the Fourth Figure assigned in the Port-Royal Logic, and by several of the German logicians, as Hoffbauer and Bachmann. Huyshe's rules are these three: 1. The Major premise must not be O, else there is an illicit process of the major; 2. The Minor premise must not be O, else there is an undistributed middle; 3. The Conclusion must not be A, else there is an illicit process of the minor.

[2] In some of the more complex of the old systems, the name of "Indirect" is given to certain moods, of which there are not here recognized any, except such as slip in under the disguise of the fourth figure. The received moods yield to them the premises without change, and the conclusion by conversion.

the given figure, are not so in the first. Thus AE, valid data in the second figure, would, in the first, yield an illicit process of the major. EA, however, are good premises in the first figure: therefore the remedy is found in the transposition of the premises, which, also, in some cases, supersedes conversion of them.

Finally, if the premises are transposed, the major and minor terms have exchanged functions. Effect must be given to this exchange by the conversion of the conclusion.

There are required, accordingly, for reduction, all the three methods: conversion of one or both premises, as a step to be taken almost always; transposition of the premises, when they would otherwise be bad; conversion of the conclusion, when the premises have been transposed.

The conclusion of the given indirect syllogism, when it has not been converted in the reduction, is proved directly by the new syllogism in the first figure. If it has been converted, there is proved, in the new syllogism, a conclusion from which the given one may be inferred by reconversion.

With these alternatives open, reduction is possible and easy, for all the indirect moods except two; AOO in the second figure, OAO in the third. The difficulty with these lies in the impossibility of directly converting O. The obstacle may be cleared away through contraposition. But an indirect reduction of another kind is also available.

The Rules of Reduction for all the indirect moods, and the relations between them and the moods of the first figure, were abbreviated by the schoolmen into clumsy names, in which certain of the letters have conventional meanings. For further aid to the memory, the names

were cast into five halting hexameters, in several readings, of which the following is one :—

I. *b*ArbArA, *c*ElArEnt, *primæ*, *d*ArII, *f*ErIO*que*:
II. *c*E*s*ArE, *c*A*m*EstrE*s*, *f*E*s*tInO, *b*ArO*c*O, *secundæ*:
III. *tertia* *d*ArA*p*tI, *d*I*s*A*m*I*s*, *d*AtI*s*I, *f*ElA*p*tOn,
*f*ErI*s*O, *b*O*c*ArdO, *habet: quarta insuper addit*
IV. *b*rA*m*AntI*p*, *c*A*m*EnE*s*, *d*I*m*ArI*s*, *f*E*s*A*p*O, *f*rE*s*I*s*On.

Of the twenty-four valid moods in figure, the scheme rejects the five which have subalternated conclusions ; and these, when they have to be referred to, may be described as Nameless Moods. The Named Moods, denoted by the leading words in the verses, are thus nineteen.

The First Figure has four named moods. Its admissible number of six is completed by two nameless moods, AAI and EAO, whose conclusions are severally subalternated from those of *Barbara* and *Celarent*.

The Second Figure has four named moods. It has two nameless moods, EAO, and AEO, standing respectively under *Cesare* and *Camestres*.

The Third Figure has six named moods. Its conclusions, being all particulars, cannot have subalternates.

The Fourth Figure has five named moods. Its one nameless mood is AEO, under *Camenes*.

The names given to the moods in the "Barbara" lines signify the rules of reduction in this way :—The vowels mark, self-evidently, the quantity and quality of the propositions constituting each mood. Of the consonants, those only are significant which are printed in italics. (1.) The *b*, *c*, *d*, and *f*, the initial letters of the four names in figure first, are the only initials used for any of the other

names. An indirect mood is thus signified to be reducible to that direct mood, whose name begins with the same letter. (2.) The other significant consonants are four: *s* directs Simple Conversion; *p* Conversion *per accidens; m* Transposition of Premises (*metathesis* or *mutatio*); *c* Reduction through Contradiction. The proposition to be operated on through *s*, *p*, or *c*, is that which is symbolized by the vowel immediately preceding the consonant: the place of *m* in the name is indifferent. The process of reduction is illustrated in the second of the annexed notes.

Note I.

EXAMPLES OF THE NINETEEN NAMED MOODS.

Each of the moods is here exemplified twice: first, by a formula framed with symbolic terms, each of which receives the signature of quantity; next, by an argument in significant terms. The matter of all the arguments is gathered from the Preface to Bishop Butler's *Sermons*. There might have been found, probably, apter instances than several of them; but brevity of expression has had to be studied.

Figure I.

1. *Barbara.*

A. All M's—are—some P's.
A. All S's—are—some M's.
∴ A. All S's—are—some P's.

A. All beings who on reflection approve virtue—are—beings conscious of obligation to act virtuously.
A. All men—are—beings who on reflection approve virtue.
∴ A. All men—are—beings conscious of obligation to act virtuously.

2. *Celarent.*

E. Any M's—are not—any P's.
A. All S's—are—some M's.
∴ E. Any S's—are not—any P's.

E. Beings conscious of law—are not—beings entitled to disobey law.
A. All men—are—beings conscious of law.
∴ E. Men—are not—beings entitled to disobey law.

3. *Darii.*

A. All M's—are—some P's.
I. Some S's—are—some M's.
∴ I. Some S's—are—some P's.

A. All beings conscious of law—are—beings liable to punishment.
I. Some living creatures—are—beings conscious of law.
∴ I. Some living creatures—are—beings liable to punishment.

4. *Ferio.*

E. Any M's—are not—any P's.
I. Some S's—are—some M's.
∴ O. Some S's—are not—any P's.

E. Beings conscious of law—are not—beings exempt from punishment.
I. Some living creatures—are—beings conscious of law.
∴ O. Some living creatures—are not—beings exempt from punishment.

FIGURE II.

1. *Cesare.*

E. Any P's—are not—any M's.
A. All S's—are—some M's.
∴ E. Any S's—are not—any P's.

E. Parts wanting mutual relation—are not—means conducing to one end.

A. All parts of a system—are—means conducing to one end.
∴ E. Parts of a system—are not—parts wanting mutual relation.

2. *Camestres.*

A. All P's—are—some M's.
E. Any S's—are not—any M's.
∴ E. Any S's—are not—any P's.

A. All parts of a system—are—means conducing to one end.
E. Parts wanting mutual relation—are not—means conducing to one end.
∴ E. Parts wanting mutual relation—are not—parts of a system.

3. *Festino.*

E. Any P's—are not—any M's.
I. Some S's—are—some M's.
∴ O. Some S's—are not—any P's.

E. Parts of a system—are not—things without a bearing on one design.
I. Some parts of a whole—are—things without a bearing on one design.
∴ O. Some parts of a whole—are not—parts of a system.

4. *Baroco.*

A. All P's—are—some M's.
O. Some S's—are not—any M's.
∴ O. Some S's—are not—any P's.

A. All systems—are—things presupposing mutual relation of parts.
O. Some wholes—are not—things presupposing mutual relation of parts.
∴ O. Some wholes—are not—systems.

FIGURE III.

1. *Darapti.*

A. All M's—are—some P's.
A. All M's—are—some S's.
∴ I. Some S's—are—some P's.

A. All beings obeying the supreme law of their nature—are—beings acting rightly.

A. All beings obeying the supreme law of their nature—are—beings acting naturally.

∴ I. Some beings acting naturally—are—beings acting rightly.

2. *Disamis.*

I. Some M's—are—some P's.

A. All M's—are—some S's.

∴ I. Some S's—are—some P's.

I. Some laws of being—are—laws of supreme obligation.

A. All laws of being—are—laws sanctioned by a penalty.

I. Some laws sanctioned by a penalty—are—laws of supreme obligation.

3. *Datisi.*

A. All M's—are—some P's.

I. Some M's—are—some S's.

∴ I. Some S's—are—some P's.

A. All judgments of reflection—are—laws superior in obligation to mere propensions.

I. Some judgments of reflection—are—judgments of conscience.

∴ I. Some judgments of conscience—are—laws superior in obligation to mere propensions.

(The truth asserted in the minor premise might have been alleged in I^2. The conversion of the I^2 into A would throw the argument into the First Figure, and justify a conclusion in A.)

4. *Felapton.*

E. Any M's—are not—any P's.

A. All M's—are—some S's.

∴ O. Some S's—are not—any P's.

E. Judgments of conscience—are not—laws affecting the individual only.

A. All judgments of conscience—are—judgments of reflection.

∴ O. Some judgments of reflection—are not—laws affecting the individual only.

5. *Feriso.*

E. Any M's—are not—any P's.
I. Some M's—are—some S's.
∴ O. Some S's—are not—any P's.

E. Beings obeying laws of their nature—are not—beings acting unnaturally.

I. Some beings obeying laws of their nature—are—beings acting rightly.

∴ O. Some beings acting rightly—are not—beings acting unnaturally.

(Here, again, the datum of the minor premise would justify an I^2, and enable us to reach, in the First Figure, a conclusion in E).

6. *Bocardo.*

O. Some M's—are not—any P's.
A. All M's—are—some S's.
∴ O. Some S's—are not—any P's.

O. Some beings obeying laws of their nature—are not—beings acting rightly.

A. All beings obeying laws of their nature—are—beings acting naturally.

∴ O. Some beings acting naturally—are not—beings acting rightly.

FIGURE IV.

(The examples in this Figure are purposely constructed with materials already used for the First).

1. *Bramantip.*

A. All P's—are—some M's.
A. All M's—are—some S's.
∴ I. Some S's—are—some P's.

(The Premises justify the conclusion, in I^2, "Some S's—are—all P's." See, in the next note, the last of the paragraphs dealing with this mood.)

A. All men—are—beings who approve virtue.

A. All beings who approve virtue—are—beings conscious of obligation to virtue.

∴ I. Some beings conscious of obligation to virtue—are—men.

2. *Camenes.*

A. All P's—are—some M's.
E. Any M's—are not—any S's.
∴ E. Any S's—are not—any P's.

A. All men—are—beings conscious of law.
E. Beings conscious of law—are not—beings entitled to disobey law.
∴ E. Beings entitled to disobey law—are not—men.

3. *Dimaris.*

I. Some P's—are—some M's.
A. All M's—are—some S's.
∴ I. Some S's—are—some P's.

I. Some living creatures—are—beings conscious of law.
A. All beings conscious of law—are—beings liable to punishment.
∴ I. Some beings liable to punishment—are—living creatures.

4. *Fesapo.*

E. Any P's—are not—any M's.
A. All M's—are—some S's.
∴ O. Some S's—are not—any P's.

E. Beings exempt from punishment—are not—beings conscious of law.
A. All beings conscious of law—are—living creatures.
∴ O. Some living creatures—are not—beings exempt from punishment.

5. *Fresiso.*

E. Any P's—are not—any M's.
I. Some M's—are—some S's.
∴ O. Some S's—are not—any P's.

E. Beings exempt from punishment—are not—beings conscious of law.
I. Some beings conscious of law—are—living creatures.
∴ O. Some living creatures—are not—beings exempt from punishment.

NOTE II.

ILLUSTRATIONS OF SYLLOGISTIC REDUCTION.

1. *Ostensive or Direct Reduction.*

1. The application of all the rules, except that indicated for two of the moods by *c*, is obvious and easy. A very few moods will suffice as examples.

FIGURE II.

The structure of this figure is such, that conversion of its major premise throws it into Figure first. That premise, being for Cesare an E, may be converted simply. We thus gain premises which, continuing to be in EA, are good for proving, in Celarent, the E of the *reducend*.

Cesare	Reduced to	*Celarent.*
The P's—are not—any M's—E.		The M's—are not—any P's—E.
All the S's—are—some M's—A.		All the S's—are—some M's—A.
∴ The S's—are not—any P's—E.		∴ The S's—are not—any P's—E.

Camestres, similarly, would fall into Figure first, if the major premise were converted. But, that premise being an A, its converse would be an I; and the resulting mood would be IE, involving illicit process of the major. If, however, the given premises are transposed, they become EA; and the premise which is now the major admits simple conversion into E. The conversion being performed, we have EA, the premises of Celarent. But the terms of the given conclusion have in these premises exchanged functions: therefore, our new conclusion must be the converse of the given one; and the new conclusion may be an E. The process, in fact, is equivalent to the transforming of Camestres into Cesare, followed by the reduction of Cesare to Celarent.

Camestres	Reduced to	*Celarent.*
All the P's—are—some M's—A.		The M's—are not—any S's—E.
The S's—are not—any M's—E.		All the P's—are—some M's—A.
∴ The S's—are not—any P's—E.		∴ The P's—are not—any S's—E.

Figure III.

Disamis is in the same predicament with Camestres; only that, the Figure being the third, the premise which would suggest itself for conversion is the minor. There would emerge bad premises in II. The remedy is found by reversing the premises, converting the I which has thus become the minor, and giving effect to the reversal by conversion of the given conclusion. The same result would follow, if Datisi were first formed from the given Disamis, and then reduced to Darii by simple conversion of the minor premise.

Disamis	Reduced to	*Darii.*
Some M's—are—some P's—I.		All M's—are—some S's—A.
All M's—are—some S's—A.		Some P's—are—some M's—I.
∴ Some S's—are—some P's—I.		∴ Some P's—are—some S's—I.

Figure IV.

Fresison has premises which, by simple conversion of both, become EI in Figure first. Therefore it is manageable by this, the process which would naturally be first thought of.

Fresison	Reduced to	*Ferio.*
The P's—are not—any M's—E.		The M's—are not—any P's—E.
Some M's—are—some S's—I.		Some S's—are—some M's—I.
∴ Some S's—are not—any P's—O.		∴ Some S's—are not—any P's—O.

Camenes, on the other hand, if so treated, would give premises in IE, yielding no conclusion. But Figure first would be gained, also, by transposition of the premises, which would then be those of Celarent. This, with the consequent conversion of the conclusion, is the method ordered.

Camenes	Reduced to	*Celarent.*
All P's—are—some M's—A.		The M's—are not—any S's—E.
The M's—are not—any S's—E.		All P's—are—some M's—A.
∴ The S's—are not—any P's—E.		∴ The P's—are not—any S's—E.

Those old logicians, who framed the name Bramantip, followed Boethius in admitting, under the name of Conversion *per accidens*, all conversions in which the *matter* allowed change of quantity, whe-

ther by particularizing or by universalizing. When we admit, as we should, no conversions but such as are correct formally, the name is deceptive: its *p* orders an operation which is impracticable. If the given conclusion is to be dealt with at one sweep, what has to be done is the conversion of I, illogically, into A. The rationale of the process (in the more obvious view of it) is this: —The premises, being transposed, become AA in Figure first. They yield, in the nameless mood AAI, a conclusion which is the simple converse of the given one. But they yield also, in Barbara, a conclusion in A. We may adopt the conclusion in A, not as holding it to be a good converse of our I, but because it follows from the premises. We do adopt it, in obedience to the principle of the scheme, that of always inferring, from premises, the widest conclusion they permit. If we still wish for the I, we may earn it lawfully by subalternation from the A.

Bramantip	Reduced to *Barbara* (or AAI).
All P's—are—some M's—A.	All M's—are—some S's—A.
All M's—are—some S's—A.	All P's—are—some M's—A.
∴ Some S's—are—some P's—I.	∴ All P's—are—some S's—A.
	(∴ Some P's—are—some S's—I.)

But Bramantip should be looked into more closely. Its premises justify a conclusion, not in I merely, but in I^2: "Some S's—are—all P's." The simple converse of this is, "All P's—are—some S's," being the A which we have just gained in Barbara. When we conclude in I from the premises of this mood, we are really inferring a subalternate of the conclusion to which we were entitled. Bramantip, however, is the only mood of the nineteen, the analysis of which requires, for its completion, any propositional form besides the received four.

2. *Indirect Reduction.*

2. The moods Baroco and Bocardo are, in the syllogistic doctrine, the *crux logicorum*. Their inflexibility arises from the double occurrence of O. The character of the conclusion compels us to look for reduction into Ferio; and this mood is not approachable through any of the methods *s*, *p*, *m*.

Those who named the two unmanageable moods, were logicians who refused to allow the conversion of O through contraposition. On this footing, it is impossible to transform either of the two into any other syllogism that shall yield, either the given conclusion, or an equivalent of it. We cannot, in the first figure, prove, from the data, that the given conclusion must be true if the premises are so. In short, that which is called ostensive reduction is impossible. But the syllogism may be reduced indirectly. We can prove, not properly in, but only through, the First Figure, that, if the premises are true, the given conclusion cannot be false.

This manner of arguing is founded on the doctrine of contradictory propositions; the two which come into play being O and A, the given conclusion and its contradictory. It is proved that, if the premises of the given syllogism are assumed to be true, the A must be false: whence, by the rule of contradictories, it is inferred that the O must be true. The only assumptions required are two, which are involved in the nature of all inference: first, that the premises of a syllogism are admitted, or are assumed to be true; secondly, that, if the conclusion of a valid syllogism be false, one or both of the premises must be false. The process of which these are the principles is called Indirect Reduction—Reduction *ad impossibile* or *absurdum*, or *ex impossibili* or *absurdo*.

The steps are the following;—The premise denoted by the vowel preceding the *c* is thrown aside; and there is substituted for it the contradictory of the given conclusion. From this new premise (an A), and the retained premise (another A), there is inferred an A in Barbara. The reduction is thus completed, according to the rules symbolized in the name.—But, if we stop here, we have proved nothing to the purpose. Our new conclusion in A has not even the same terms with the given conclusion in O. We have promoted our given middle term to the rank of major in dealing with Baroco, to that of minor in dealing with Bocardo. In effect, the syllogism in Barbara has merely supplied one datum for a process of inference through contradiction, in which the essential part of the operation lies.

The A which is the conclusion in Barbara is found, on compa-

rison, to be the contradictory of the O which was the premise elided from the given syllogism. But this O, as a datum, must be assumed to be true: therefore the A must be false. Now, the A has been inferred validly from its premises: therefore, one or both of these must be false. The premise borrowed from the given syllogism must, as a datum, be true: therefore the falsity must lie in the premise which we constructed for ourselves. But this premise is the contradictory of our given conclusion; and, the contradictory being false (having been proved to be inconsistent with the original data), the given conclusion must be true. Q. E. D.

Baroco	Reduced to	*Barbara.*
All P's—are—some M's—A.		All P's—are—some M's—A.
Some S's—are not—any M's—O.		All S's—are—some P's—A.
∴ Some S's—are not—any P's—O.		∴ All S's—are—some M's—A.

A, the conclusion in Barbara, is the contradictory of O, the rejected premise in Baroco: the O being assumed as true, the A must be false. Consequently, one or both of the premises of the Barbara must be false. Its major premise must be assumed as true, having been a premise in the given Baroco: therefore the minor premise must be false. But this minor premise is the contradictory of the given conclusion; therefore the given conclusion must be true.

Bocardo	Reduced to	*Barbara.*
Some M's—are not—any P's—O.		All S's—are—some P's—A.
All M's—are—some S's—A.		All M's—are—some S's—A.
∴ Some S's—are not—any P's—O.		∴ All M's—are—some P's—A.

The subsequent inferences proceed, step by step, as for Baroco.

3. The demonstration that a proposed conclusion must be true, because it is impossible that it can be false, is not only always cumbrous, but also, in many instances, slow in generating conviction. It is never used, by logicians or by others, when it can be escaped from. Yet it is of common use and very wide applicability. Even axioms, whether popular or philosophical, may have their meaning, if not their truth, brought out more clearly, when it is shown that the denial of them must lead to consequences which

are absurd. In controversy, demonstration through impossibility lies at the root of the argument *ad hominem:* an opponent denying a conclusion of ours, we endeavour to prove that his denial is contradictory of propositions which he himself admits. In geometry, indirect demonstration is familiar; and the name, *Reductio ad absurdum,* which mathematicians are wont to give it, comes from its syllogistic application, and is needlessly retained when reduction is not aimed at.

It may be worth while to observe two points of difference, between the logical uses of indirect demonstration on the one hand, and the scientific and popular uses of it on the other.

First, In the search for propositions whose truth is inconsistent with the falsity of a proposition to be proved, an expounder of the exact sciences has before him a field of choice broadening with every truth he has already been able to prove. The impossibility of the falsehood of a theorem is demonstrable through the inconsistency of the falsehood with any of the truths which, whether as axioms or as propositions proved, have previously received a place in the system. Even in questions involving contingent truths the position of a reasoner is not dissimilar to this.—Contrariwise, in the indirect reduction of a syllogism, the logician can assume nothing except his two premises. But, as we have learned, nothing else is necessary. The logical procedure is, in fact, with the one exception of its working towards the first figure, the very same process which is and must be followed, in all demonstrations through contradiction. The search for data is only a gathering of the materials, out of which is woven a chain of reasoning, every link of which depends on the law of contradiction, and is transmutable into the strictest logical forms.

Secondly, In mathematics always, in other kinds of matter not seldom, the inconsistency founded on lies between propositions logically describable, not as contradictories, but only as contraries: that is, it lies between E and A. Why, it may be asked, is this sufficient? For plain reasons, indicating very frequent cases. In the first place, contraries cannot both be true: if we can hold the one to be true, we may infer the falsity of the other. Again, in the exact sciences,

it is evident that no propositions are particular: all are either universal or singular. Therefore the law of contradiction presses from all sides on the contraries, with the same force as on contradictories when they are possible. This remark might be stretched very far indeed beyond the bounds of the sciences treating number and quantity. All philosophical doctrines, whether of mind or of matter, are strictly universals. But the illustration of this point would lead us much out of our way.

3. *Reduction through Contraposition.*

4. Contraposition, and Conversion of the Contraposita, being allowed for O and A, Baroco and Bocardo may be reduced directly or ostensively.

The principle of the process appears most clearly, when we suppose the contraposition to be first completed, to the effect of throwing the syllogism into another mood in the same figure. In the second figure, our negative conclusion must be retained: therefore, for Baroco, we must transform the AO of the premises into EI. The syllogism thus passes into Festino. Indeed, in framing a syllogism from given materials, it is often a matter of indifference which of these two moods we throw it into. The relation between Baroco and Festino is paralleled, in the third figure, by that between Bocardo and Disamis: the transformation is here effected by transforming each of the O's into an I. The syllogisms thus framed, in Festino and Disamis, are then subject to the ordinary rules of Ostensive Reduction.

Some of the logicians who admit this method, have emulated the schoolmen in the fabrication of names, designed to intimate, like the old ones, the manner of performing the process at one step. In these names, the *c* denotes conversion, through contraposition, of the preceding proposition: *m* denotes transposition of premises as before. In this way, Baroco is represented by Facoro, and Bocardo by Docamo. But these names, though adopted by Whately, fail to point out all the steps. If we must have names, exactness would require such as these still more whimsical ones—Facoco, Docamoc.

Baroco, thus treated, gives a syllogism in Ferio, proving the

given conclusion. Bocardo gives a syllogism in Darii, proving the converse of the contraposita of the given conclusion.

The reduction, whether taken at one step or in two, requires no illustration beyond the examples.

BAROCO—Reduced (through Festino) to—FERIO.

1. *Baroco.*	2. *Festino.*
All P's—are—some M's—A.	The P's—are not—any things not M's—E.
Some S's—are not—any M's—O.	Some S's—are—some things not M's—I.
∴ Some S's—are not—any P's—O.	∴ Some S's—are not—any P's—O.

3. *Ferio.*

Things not M's—are not—any P's—E.
Some S's—are—some things not M's—I.
∴ Some S's—are not—any P's—O.

BOCARDO—Reduced (through Disamis) to—DARII.

1. *Bocardo.*	2. *Disamis.*
Some M's—are not—any P's—O.	Some M's—are—some things not P's—I.
All M's—are—some S's—A.	All M's—are—some S's—A.
∴ Some S's—are not—any P's—O.	∴ Some S's—are—some things not P's—I.

3. *Darii.*

All M's—are—some S's—A.
Some things not P's—are—some M's—I.
∴ Some things not P's—are—some S's—I.
∴ Some S's—are—some things not P's—I.
∴ Some S's—are not—any P's—O.

4. *Wider Applicabilities of Reduction.*

5. If the Reduction of syllogisms were merely a game to be played at (and sometimes it has been elaborated till the practical sinks out of sight), the ball might evidently be tossed from any of the four quarters of the field, and pass from any one hand into any other. It would be difficult to exhaust, for any given syllogism, the possibilities of metamorphosis through such operations as the rules of reduction allow.

In transferring imperfect syllogisms to the first figure, the quality of the propositions would raise no very strong barrier between the affirmative moods and the negative: the quantity of the terms would fix the only peremptory limit. Reduction *ex impossibili* is as apt, and as easy, for any other moods, as for the two which have a monopoly of it; although, for some of them, we should have to be content with arguing from the contrary instead of the contradictory. Further, moods of any figure might be changed into moods of any other.

The scrutiny of syllogisms for such purposes as this, does, in more instances than one, point to curious relations between moods, all of them traceable ultimately to the fundamental laws. The inquiry might be made, in logical teaching, a useful discipline of sagacity and exactness; and it might also draw the study of the syllogism away from that mechanical reliance on rules, into which the very symmetry and completeness of the scholastic system are apt to make it degenerate.

But the results have no claim to reception in an outline of logical science.

DIVISION II.—THE SYLLOGISM ANALYSED IN EXTENSION AND COMPREHENSION.

The bearing of the wholes of predication on the structure of the syllogism.

88. If we were to be content with possessing a set of rules enabling us to test, easily and infallibly, the conclusiveness of every argument that could be proposed, the syllogism would have been dissected deeply enough, when it had yielded the two canons, and the corollaries flowing from them. Differences of figure would have no importance: the process of reduction and its principle might, with equal safety, have been left unexamined. If, again, seeking for a higher law, we still aimed only at forcing the indirect moods into the precinct over which the *dictum* bears sway,

the end would have been attained when the study of the scholastic scheme had put it in our power to transform every imperfect syllogism into one of the perfect forms which are to be found in the first figure.

But, while the canons, and the rules of reduction, have each a substantive logical value, the syllogism is most incompletely understood, if it is studied only through either of those media, or even through both. The former, taken alone, would leave us where we should be left, in biological science, if we had qualified ourselves only for distinguishing and describing a part of the animal body, without having learned anything as to the physiological relations between the given object and others. By the latter method, on the other hand, we should, indeed, begin the study of functions; but we should drop it before having gone further than the collection of data: our position would be that of the anatomist, who, after having microscopically determined the structure of the animal tissues, should refrain from asking how that structure affects the action of the organs which the tissues constitute.

Reasoning through common terms has pre-formed classifications as its data. It is merely an explication, an evolution into the form of judgments and propositions, of relations implied in a presupposed ordination of the concepts and terms.

What the ordination is, that is presupposed in a given process of mediate inference, we ascertain at a glance when the syllogism is in the first figure. In that figure we infer, in regular descent of extension, from term containing to term contained; from the major, through the middle, to the minor. This order of the terms is necessarily, for the first figure, the order of extension assumed

in the pre-formed series of terms. The order of comprehension is, of course, that order reversed.

What the presupposed ordination of terms is, in a syllogism in any of the indirect moods, is learned through the relation of that mood to a mood in the first figure. That relation has been exhibited to us, for all the indirect moods, in the Rules of Reduction. Though reduction should in itself be held worthless, its rules are invaluable, as facilitating progress to something beyond them. They are premises, regularly and completely arranged, from which may be inferred, more easily by far than if we wanted them, the true relations of the several syllogistic figures, the truths in virtue of which each of the figures has a place, though not all a place equally high, in the actual system of human intelligence.

The rules bridge over, for occasional passage, the gap which yawns between the first figure and the others. Where their foundations are dug into, modes of thought are laid bare, which unite all the figures into one symmetrical formation.[1]

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[1] The principle which it is here attempted to bring to bear on the syllogism, is that whose positive applicability to such a use may fairly be said to have been undreamt of till it was declared by Sir William Hamilton, and which has hitherto been so applied by him alone. (See Note to Section 33.) Those students of the science (and they do seem to be as yet few) who, in their inspection of Hamilton's logical system, have advanced, from his doctrine of quantification, to this deeper-lying section of his theory, will perceive already, that the purpose for which the principle is here used is different from his;—if, indeed, those scanty notices, which only have hitherto been made public, entitle us to judge precisely as to his views of the ultimate bearing of the principle.

Both in the *New Analytic*, and in Hamilton's own Appendix to

The differences in the character of predication between the first figure and the other three.

89. Two preliminary points require to be considered, for enabling us to extricate, out of the received syllogistic doctrines, all the relations of the syllogism, both to the extension and to the comprehension of its three terms.

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the *Discussions*, the point which comes out most prominently is, the reducibility of all the three receivable syllogistic figures under one law. The characteristic differences of the three figures, as traceable to the wholes in which their propositions severally predicate, are, indeed, exhibited to demonstration; but reduction of the second and third figures to the first seems to be condemned, as a process both needless and unscientific. The opinion which has forced itself on the present writer is this: that the first figure, as founded on the ordinated hierarchy of the three terms, while the others are not so, must retain its Aristotelic pre-eminence; that the functions of the other figures, as expressions for processes of thought, do not appear clearly unless through comparison with the first figure; that reduction by the common rules gives the only means of such a comparison; and that the view here taken of conversion, as a transference of predication from whole to whole, reconciles reduction completely to the true theory of the relation between extension and comprehension.

Trendelenburg, it was already observed, asserts peremptorily the necessity for considering both wholes, if we are to have a philosophical theory of the syllogism. But he treats the question quite generally, and also (it may rightly be said) no more than negatively. Even here, he stands much closer to Schleiermacher than to Hamilton. He maintains the insufficiency of the analysis in extension; but, taking his position as an out-and-out adversary of the Formal Logic, he abstains from all attempts at positive application of the principle to the formal syllogistic scheme. Indeed, his comparison of the wholes leads him, at once, on to that objective ground, on which he aspires to founding logical science. That the distance between his point of view and Hamilton's may be the more clearly seen, there are here set down, from his *Logische Untersuchungen*

In the first place, the forms of predication, on the analysis of which the received system rests, are those in which the terms are concrete, not abstract. It is, in fact, all but

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(sect. xvi., vol. ii.), a few of those passages in which his opinions appear with the smallest admixture of the metaphysical element.

After having observed that the usual form of the "Dictum" contemplates extension, and that the "Nota notæ" contemplates comprehension, he proceeds thus: "The syllogism emerges out of the reciprocal reference of comprehension and extension. If the comprehension of a concept (the positive or negative law) is applied to its extension, there arises the *categorical* syllogism. The comprehension (major term) of a concept (middle term) governs its extension (any or all of the species, minor term). If, on the other hand, the same law is expressed for all the species, and if, out of this content (or comprehension) of the extension, the comprehension of the containing universal is collected, there arises the *disjunctive* syllogism. The species constitute the middle term, or concept, whose comprehension becomes the comprehension of the genus." (Pp. 239, 240). . . . . "Comprehension and extension, in the relation of Law and Phenomenon, constitute the essential elements (*Seiten*) of the concept; and their reciprocal relation constitutes its life." (P. 241). . . . . "What vouches, then, for the completeness of the forms of the syllogism? The comprehension is referred to the extension; and out of the extension the comprehension is determined, and that both positively and negatively." (P. 248). . . . . "Must the syllogism, then, be nothing but a subjective function without a real counterpart? No. The comprehension, representing the law of the extension, contains the possibility of the syllogism; and therein also is its objective value intimated. To the *genetic* universal, which rests on an original community of thought and existence, there corresponds the *quantitative* universal. The necessary ground (reason) hence clothes itself in the expression of a universal fact, and becomes, in this form, the middle concept of an objective syllogism. That which, in the real, is the ground

impossible, as we have already seen, to express intelligibly differences of quantity, when abstract terms are adopted throughout; while, likewise, the union of concrete terms with abstract, both betrays and causes confusion of thought. Concrete terms, however, point directly to predication in extension; and this is one of several reasons why the relation of comprehension has been so much overlooked.

Secondly, the conversion of a proposition is nothing else or more than the transference of predication from the one whole into the other. When this doctrine is called to mind, it becomes evident how thoroughly the relations of the figures to the wholes are implied in the rules of reduction.

These considerations being premised, it is the easiest thing in the world to mark, for all syllogisms whose propositions are either A, E, I, or O, the whole in which every one of the propositions predicates.

Considered without reference to the wholes of predication, the three receivable figures severally exhibit the three terms as formally ordinated in this way. In figure first, the middle stands between the minor and the major; in figure second, the middle stands above both of them, the one being predicated in, the other out of it; in figure third, the middle stands below both of them, being predicated in both, or in the one and out of the other. When we have dis-

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(reason), becomes, in the logical, the middle term of the syllogism. Aristotle himself acutely pointed out this parallelism. (*Analyt. Post.* ii., 2, 11, 12; *De Anima*, ii., 2: compare Trendelenburg's *Elementa Logices Aristoteleæ*, § 58, &c.) But the formal logic, which would have nothing to do with the real, allowed this profound suggestion to lie unused." (P. 280).

covered in which of the wholes it is that each of the predications takes place, we have discovered also how it is, that, in the second and third figures, the middle term has come to hold a position different from that which should belong to it, as being the connecting link of thought between the minor and the major.

I. In the first figure, each of the three propositions is a Predication in Extension. The minor term is placed in the extension of the middle, the middle in the extension of the major, or of its contradictory; and, consequently, the minor is placed in the extension of the major, or of its contradictory.

Clearly, too, by converting all the three propositions, we should gain three propositions in comprehension. If, indeed, we are prohibited from adopting any propositional forms except the received four, the syllogism formed must be bad. But if the conversion were made thorough, through additional forms, the new syllogism would not violate, either the law of mediate inference, or any of the common rules. This view, however, is neglected in the books.

In a word, the first figure predicates exclusively in one whole.

II. None of the other three figures predicates exclusively in one whole. In each of them, two of the propositions are predications in the one whole; while the third is a predication in the other.

The several indirect moods stand in two several relations to the first figure, indicated respectively by their requiring or not requiring, in reduction, transposition of premises, and consequent conversion of the conclusion. This difference makes it impossible to lay down, for syllogisms

expressible through the four received forms of predication, laws systematically distinguishing each of the indirect figures from the others. But an approach is made to such laws in the following statement:—

Figure II.

In *Cesare*, *Festino*, and *Baroco*, the major premise predicates in comprehension; the minor and the conclusion predicate in extension. In *Camestres*, whose premises in this order are bad for the first figure, the major premise predicates in extension; the minor and the conclusion predicate in comprehension. In this figure, for all moods, the whole in which the minor premise predicates, determines the whole predicated in by the conclusion.

Figure III.

In *Darapti*, *Datisi*, *Felapton*, and *Feriso*, the minor premise predicates in comprehension; the major and the conclusion predicate in extension. In *Disamis* and *Bocardo*, whose premises (equivalents through contraposition) are in this order bad for the first figure, the minor premise predicates in extension; the major and the conclusion predicate in comprehension. In this figure, for all moods, the whole in which the major premise predicates, determines the whole for the conclusion.

Figure IV.

In *Bramantip*, *Camenes*, and *Dimaris*, both premises predicate in extension; the conclusion predicates in

comprehension. In *Fesapo* and *Fresison*, both premises predicate in comprehension; the conclusion predicates in extension. In the moods whose premises must be transposed to pass into the first figure, the conclusion is in comprehension; in those whose premises do not require transposition, the conclusion is in extension.

The predications of the first figure analysed in extension.

90. The general character of the First Figure has already come out so clearly, as to call, now, for no special explanation.

It is distinctively the form of Deductive Thinking. It presupposes the classification of objects in the progressive order of generalization; and, starting from the widest class, it infers, through a class intermediate, the inclusion of a lowest class within the sphere of the highest.

A principle or law, assumed as already established, is inferred to govern or not to govern a given case, because it is asserted that the given case is included in a class of cases which are known to be governed or not to be governed by the assumed law or principle. The given case S is included in the class of cases M; the whole class of cases M is governed by the law P or Not-P: therefore the case S is governed by the law P or Not-P. Or, otherwise, for negatives, the case S is included in the class of cases M; the whole class of cases M is excluded from the cases governed by the law P: therefore the case S is excluded from the class of cases governed by the law P. The ordination of terms is, always, from most to least extensive, P (or Not-P), M, S.

There thus appears, first of all, the distinction between the two Affirmative moods of the figure, *Barbara* and *Darii*, and the two Negative moods, *Celarent* and *Ferio*. The difference arises wholly out of the quality of the assump-

tion made in the major premise. That premise determines the conclusion to affirmation or to negation. When it is affirmative, the inclusion of the terms is direct in all its three steps. When it is negative, the inclusion in the second step, and, consequently, that in the third, may still be held to be a direct inclusion in the contradictory of the major. But the laws of ordination would allow us, also, for negatives, to hold the second step as being an exclusion of co-ordinates (M excluded from the extension of P), and the third step, consequently, as an exclusion of a subordinate from a co-ordinate of its superordinate (S excluded from the extension of P).

Next appears the distinction between the two Universal moods, *Barbara* and *Celarent*, and the two Particular moods, *Darii* and *Ferio*. This difference arises wholly out of the quantity of the subsumption made in the minor premise. According as all of the S's, or only some of them, are placed in the middle term, so all, or only some, are inferred to be in, or out of, the major. The difference may, in effect, be considered as accidental: it arises properly out of our poverty in class-names. The objects designated by our minor term are for us a whole: they are a whole which we are not compelled to regard as constituted by parts: they are our lowest datum, which must pass, through the middle term, into or out of the major, without change or analysis. In this figure, indeed, the minor term may be a singular, without any impinging either on principle or on form. If we give to the minor term the signature "some," this is only because we have not a name narrow enough to yield "all:" we cannot specificate the objects more exactly than by saying, that they are "some" of the objects constituting the class S. If we were to describe them as "All

the objects which are called some S's," we should have ground for a universal conclusion: and this suggestion may help to show how these particular moods are really inessential variations of the universal ones.[1]

The predications of the second figure in both wholes.

91. The mutual dependence of comprehension and extension makes inference to be possible, though the deductive sequence of the terms is departed from. But the departure imposes limitations, either on the quality of the conclusion or on its quantity.

When the sequence is deserted in the premise which either is the *major* as given, or would be the major if the sequence were obeyed, the conclusion is made *negative*. This is the predicament of the Second Figure. Its characteristic shortcoming, when looked at from the objective side, lies in this. While it asserts, in its lowest step, the minor premise, that S is one of the class of cases M, it does not assert, in the major premise, either that the class of cases M is included under, or excluded from, the operation of

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[1] This universalizing of the minor term would be useful, also, if we were to attempt expressing Ferio in comprehension. It would enable us to dispense altogether with Hamilton's partial negatives. It serves the same use in the second and third figures, for moods which have the conclusion in O. For the universalizing may be used, in the indirect figures, not indeed always for the minor term, but always for the term which would be the minor if the syllogism were reduced to Figure first. Festino is thus manageable directly, Baroco through it, but not otherwise; Felapton and Feriso directly. Bocardo cannot discard the partial negatives in comprehension, unless by being transformed into the affirmative mood Disamis: Fesapo is independent of them, without being universalized.

the law P. Hence there are no data for affirmation: and it is only indirectly that even negation is justified.

I. The normal character of the figure is exhibited by *Cesare*, and its subalternate mood *Festino*. In the minor premise of these, the foundation is laid for a deductive argument. The minor is placed in the extension of the middle. But there does not follow a major premise, which shall place the middle term either in or out of the extension of the major. We turn abruptly round, and predicate in comprehension. The placing of the major in the comprehension of the middle would plainly determine nothing, the middle having in the other premise been considered only in part. Therefore we exclude the major term from the comprehension of the middle.

II. The necessity of transforming *Baroco* into *Festino*, through contraposition, if it is to exhibit the ordination of terms in any workable shape, is prognosticated by its initial step. Its minor premise excludes the minor from the extension of the middle, and thus seems to bar all further progress.

III. *Camestres* is merely *Cesare*, with the premises misplaced, and with the legitimate functions of the minor and major terms consequently interchanged.

The predications of the third figure in both wholes.

92. The position of the Third Figure is this. It deserts the sequence of deduction in its initial step: its *minor* premise, or that which is the minor when the deductive sequence is established, predicates in comprehension. The consequent limitation of the result falls on the quantity: the conclusion is necessarily *particular*.

The root of the restriction lies in the opposite quarter from that which made the second figure negative. The

compass of the law P is here definitively fixed, either positively or negatively; but the cases to be placed within the law, or out of it, are assumed as being only a part of S. The reason is, that, in our lowest step, we want the clew to the ordination. We are not able to assert, in our minor premise, that the S's, or any of them, are in the class of cases M: we are able to assert, only, that the cases M, or some of them, possess the attribute S; and, not holding ourselves entitled to aver that they are in exclusive possession of that attribute, we say only that the M's, or some of them, are some of the S's. Accordingly, when, in our major premise, all the M's have been asserted to be under, or free from, the law P, it is only of some of the S's that we can infer the subjection to the law, or freedom from it.

The moods of this figure, when they are examined with reference to the deductive sequence of terms, and the consequent relations to the first figure, are found to fall into three classes. In scrutinising all of these, we must remember that, as a consequence of the imperfection of knowledge implied in the lower of the premises, the minor term must pass through the syllogism unchanged, as "some S's."

I. The character of the figure appears most purely in *Datisi*, and the parallel negative, *Feriso*. The particularity of the minor term being kept in view, the terms appear as rising by steps, each of which is a part only of the higher. Our "some S's" are a part of the class M; the whole class M is a part of the class P (or Not-P): consequently the "some S's" are a part of the class P (or Not-P). The data do not exceed the minimum required for justifying the result.

II. In *Darapti*, and its negative *Felapton*, the data do exceed the minimum; for, though the subsumption is wider

than in the first two moods, the conclusion is not so. When all the propositions have been made predications in extension, their import, quantities being fully expressed, is this: "Some S's are *all* M's; all M's are some P's (or Not-P's): therefore some S's are some P's (or Not-P's)." The class which, for want of a more precise name, we call "some S's," is not, as before, a part of the class M: it is identical with that class, or constitutes the whole of it. The subordination of the minor term to the middle is merely formal; and even the appearance of subordination vanishes when the minor premise is fully converted. The ordination presupposed is not genuine. "Some S's" and "all M's" are equipollent terms, identical both in extension and in comprehension.

III. *Disamis*, and its negative *Bocardo*, are in the same situation as *Camestres*. *Disamis* takes a circuitous path to reach the conclusion of *Datisi:* and *Bocardo*, going still further astray, by starting from a negation in its lower premise, cannot be brought to exhibit the deductive relations of its terms, otherwise than by a process which is equivalent to its passage through *Disamis*.

The predications of the fourth figure in both wholes.

93. The rules for reduction exposed the Fourth Figure, as being a deformed variety of the first. Inspection of the wholes of predication at once renders the reason of the rules, and shows the source of the clumsiness.

The five moods fall obviously into two classes, differing diametrically in the character of their deviations from the deductive progress.

I. In *Bramantip*, *Camenes*, and *Dimaris*, we have received good premises for *Barbara*, *Celarent*, and *Darii:* and we retain these as predications in extension. But we

misplace them ; or, without misplacing, we mistake the functions of the major and minor terms. From our premises in extension, we draw, by a sudden inversion, a conclusion in comprehension.

II. In *Fesapo* and *Fresison*, both premises predicate in comprehension. These would yield, also, in comprehension, good conclusions, but only in $\frac{1}{2}$E, unless we were to universalize the minor term. The conclusions which we do infer are predications in extension. *Fesapo*, again, is, in respect of ordination, situated similarly to *Felapton*.

It is plain that, in neither of the classes, does the character of the premises present any feature entitling the figure to be ranked as natural or independent. In the moods of the first class, we have simply misused premises which are good for the first figure, and which are expressed in a form fitting them for convenient use. In the moods of the second class, the difficulty of managing the given predications in comprehension does make it more difficult, but still not impossible, to reach the pure deductive sequence. If the figure had any claim to a place in the legitimate system, it could only be in respect of these two moods, or rather in respect of *Fresison* alone.

The transformability of all syllogisms by exhaustive conversion.

94. A Proposition must be a Predication in one Whole. But every proposition may, through conversion, become a predication in the whole opposite to that in which it was given. Consequently, every one of the nineteen or twenty-four syllogisms of the scholastic scheme might be made to assume a new form, by having each of its three propositions converted.

The new syllogisms, however, would correctly represent the

old, on this condition only, that the conversion should everywhere be not only safe but exhaustive. Not only must no term be distributed in the converse, which was undistributed in the convertend; but, also, no term must be undistributed in the converse which was distributed in the convertend. Clearly, if this requirement were not complied with, the converted syllogisms might, through failure of distribution, contain fallacies of quantity not committed in the syllogisms from which they had been formed.

The condition, however, cannot be fulfilled through methods of conversion obeying the common rules, or confined to the four propositional forms A, E, I, O.

Ordinary speech, like ordinary thinking, proceeds by preference in the whole of extension: we never do, continuously or systematically, either think or speak in comprehension. The received doctrine of predication, and the syllogistic theory which assumes it, are alike accommodated to this universal tendency. The four propositional forms are designed, and are adequate, to express processes of thought, which are conducted explicitly in extension, comprehension being only silently implied. The mediæval theory of the categorical syllogism, in which the doctrine of predication receives its highest development, takes for granted predication in extension only. Accepting the first figure as the norm, it lays down, indeed, general principles, which are correctly applicable to propositions expressed in either whole: but its whole array of specific rules is worked out on the supposition, that the propositions of that figure predicate in extension, and not otherwise.

In every one of the indirect moods there are, as we have discovered, propositions which, extension being accepted as the form of the first figure, must be interpreted as pre-

dications in comprehension, in virtue of that ordination of terms, of which the first figure is an explication. No serious difficulty arises hence, so long as we rest satisfied with treating syllogisms in the scholastic fashion. Almost all those propositions in comprehension, which appear in any of the indirect moods, are expressible in one or another of the four established forms: indeed, the genuine conclusion of *Bramantip* is the only exception. Even though we pass to reduction, those forms are sufficient for exhaustive conversions in all cases, except the minor premises of the three anomalous moods, *Darapti*, *Felapton*, and *Fesapo*.

If, however, we aim at a conversion of *all* the propositions of a syllogism, there is not found, among the fourteen named moods of the three genuine figures, any one for which the A, E, I, O are sufficient. The I^2 is peremptorily called for as expressing the only thorough converse of A. Even the $\frac{1}{2}$E, too, would have to be taken as the converse of O if conversion of it were insisted on. This weak form is indeed avoidable, through universalizing of minor terms, for all moods except *Baroco* and *Bocardo:* but for these it is inevitable, unless we hold (as we ought) that they are not directly expressible through conversion.

To a series of syllogisms thus made up, almost all the specific rules of the common system would clearly be inapplicable. But it is a test of the soundness of the orthodox system, so far as it goes, that all the principles on which its rules are founded tell on the transformed syllogisms with undeviating exactness.

95. The possibility of thus setting forth any syllogism of the common scheme in each of two convertible shapes, is a point of doctrine for which we cannot be too thankful.

The predications of the first figure analysed in comprehension.

It is at once a decisive proof, and an instructive illustration, of that theory of the counter-relations connecting the wholes of a concept, without which the laws of the syllogism are left destitute of a firm philosophical foundation.

But nothing of a practical applicability seems to be attainable through the exhaustive transformation of the received moods. It is enough, at all events, for the purpose here in view, that the process be exemplified by exhibition of the forms which the Named Moods of the First Figure put on, through exhaustive conversion of all their propositions.

The premises in extension are here set down in the usual order. For expression of the syllogism in comprehension, they must of course be transposed; since the conversion of the conclusion makes its terms exchange functions.

BARBARA.

1. *In Extension.*

Major Premise .... All M's—are—Some P's (A).
Minor Premise .... All S's—are—Some M's (A).
Conclusion .... ∴ All S's—are—Some P's (A).

2. *In Comprehension.*

Major Premise .... Some M's—are—All S's (I^2).
Minor Premise .... Some P's—are—All M's (I^2).
Conclusion .... ∴ Some P's—are—All S's (I^2).

CELARENT.

1. *In Extension.*

Major Premise .... Any M's—are not—Any P's (E).
Minor Premise .... All S's—are—some M's (A).
Conclusion .... ∴ Any S's—are not—any P's (E).

2. *In Comprehension.*

Major Premise .... Some M's—are—all S's (I^2).
Minor Premise .... Any P's—are not—any M's (E).
Conclusion .... ∴ Any P's—are not—any S's (E).

DARII.

1. *In Extension.*

Major Premise .... All M's—are—some P's (A).
Minor Premise .... Some S's—are—some M's (I).
Conclusion .... ∴ Some S's—are—some P's (I).

2. *In Comprehension.*

Major Premise .... Some M's—are—some S's (I).
Minor Premise .... Some P's—are—all M's (I^2).
Conclusion .... ∴ Some P's—are—some S's (I).

FERIO.

1. *In Extension.*

Major Premise .... Any M's—are not—any P's (E).
Minor Premise .... Some S's—are—some M's (I).
Conclusion .... ∴ Some S's—are not—any P's (O).

2. *In Comprehension.*

Major Premise .... Some M's—are—some S's (I).
Minor Premise .... Any P's—are not—any M's (E).
Conclusion .... ∴ Any P's—are not—some S's ($\frac{1}{2}$E).[1]

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[1] The Toto-Partial Negative in *Ferio* is avoidable, if, adopting a hint already thrown out, we read the mood as *Celarent*, by interpreting the minor term as distributed: "All the things we call some S's." Its transference to comprehension then yields again I^2EE.

If, throwing aside figure fourth, we do thoroughly convert all the propositions of all the fourteen named moods in the others, we

double our list of admissible moods. The twenty-eight thus appearing are just so many of Mr Thomson's or of Sir William Hamilton's. It may be convenient, both for students of those systems, and for any who may wish to test minutely the suggestions here offered, that the twenty-eight be summarily identified with the corresponding moods of those authors. Mr Thomson (p. 248), has tabularized all Sir W. Hamilton's moods, translating the names into his own symbolic letters. A, E, I, O, retain their old significations: our A^2, Hamilton's toto-total affirmative, is Thomson's U; our I^2, the parti-total affirmative, is his Y; our $\frac{1}{2}$E, the toto-partial negative, is η; our $\frac{1}{2}$O, the parti-partial negative, is ω. Our own symbols, here to be repeated, are easily interpretable into either of the other sets.

The formal outline of Hamilton's scheme is this. The first three figures only are admitted. The eight possible forms of predication being accepted, there are, in each figure, twelve valid affirmative moods. Each of these yields in each figure two negative moods: No. 1, by making the major premise negative; No. 2, by making the minor premise negative. There are thus, in each of the three figures, 36 moods: 12 Affirmative, and 24 Negative. Sir W. Hamilton's moods are, accordingly, 108 in all. Mr Thomson reduces the number to 63, by rejecting 45 moods, which introduce $\frac{1}{2}$E or $\frac{1}{2}$O, or both.

In identifying the moods, we must bear in mind certain consequences of the conversion. By converting the conclusions, we make transposition of premises necessary for all the three admitted figures. The conversion, also, throws the Second Figure into the Third, and the Third into the Second.

In the Fourth Figure, conversion of all propositions would only displace one anomaly to make room for another. Not recognised in the new scheme, it can be treated only by having all its predications thrown into the same whole, and the premises arranged for the first figure. If they are all expressed in extension, we gain, except for one mood, named syllogisms of the common scheme; and its moods, so treated, may then be transferred, if we will, to comprehension.

In the following summary, the Named Moods, and their conversive equivalents, are referred to the numbers which the Affirmatives bear in Mr Thomson's table, from i. to xii., the Negatives under each being noted as 1 and 2.

THE NINETEEN MOODS.

FIGURE I.

1. *Common Form.*

1. BARBARA... ......AAA — No. iii.
2. CELARENT ....EAE — From No. ix., A^2AA : Neg. 1.
3. DARII ....AII — No. v.
4. FERIO ....EIO — From No. xi., A^2II : Neg. 1.

2. *Converted Form.*

1. BARBARA..........$I^2I^2I^2$ — No. iv.
2. CELARENT ....I^2EE — From No. x., $I^2A^2I^2$: Neg. 2.
3. DARII ....II^2I — No. vi.
4. FERIO ....$IE\frac{1}{2}E$ — From No. xii., IA^2I : Neg. 2.

FIGURE II.

1. *Common Form.*

5. CESARE ....EAE — From No. ix., A^2AA : Neg. 1.
6. CAMESTRES.......AEE — From No. x., AA^2I : Neg. 2.
7. FESTINO...........EIO — From No. xi., A^2II : Neg. 1.
8. BAROCO ....AOO — From No. iv., AI^2I^2 : Neg. 2.

2. *Converted Form; yielding Figure III.*

5. CESARE ....I^2EE — From No. x., $I^2A^2I^2$: Neg. 2.
6. CAMESTRES ....EI^2E — From No. ix., A^2I^2A : Neg. 2.
7. FESTINO ....$IE\frac{1}{2}E$ — From No. xii., IA^2I : Neg. 2.
8. BAROCO ..,.......$\frac{1}{2}EI^2\frac{1}{2}E$ — From No. iii., AI^2A : Neg. 1.

Figure III.

1. *Common Form.*

9. Darapti ....AAI — No. ii.
10. Disamis ....IAI — No. vi.
11. Datisi...... .....AII — No. v.
12. Felapton.... ...EAO — From No. vii., A^2AI : Neg. 1.
13. Feriso ....EIO — From No. xi., A^2II : Neg. 1.
14. Bocardo.........OAO — From No. iv., I^2AI^2 : Neg. 1.

2. *Converted Form; yielding Figure II.*

9. Darapti.........I^2I^2I — No. ii.
10. Disamis ....I^2II — No. v.
11. Datisi ....II^2I — No. vi.
12. Felapton ......$I^2E\frac{1}{2}E$ — From No. viii. I^2A^2A : Neg. 2.
13. Feriso ....$IE\frac{1}{2}E$ — From No. xii., IA^2I : Neg. 2.
14. Bocardo ....$I^2\frac{1}{2}E\frac{1}{2}E$ — From No. iii., I^2AA : Neg. 2.

Figure IV.

1. *Common Form.*

15. Bramantip ...AAI(rather I^2) — Unacknowledged.
16. Camenes.......AEE — Unacknowledged.
17. Dimaris ....IAI — Unacknowledged.
18. Fesapo ....EAO — Unacknowledged.
18. Fresison........EIO — Unacknowledged.

2. *Re-arranged Form; yielding Figure I. in Extension.*

15. Bramantip ...AAA — No. iii. (=Barbara).
16. Camenes.........EAE — From No. ix., Neg. 1 : (=Celarent).
17. Dimaris ....AII — No. v. (=Darii).
18. Fesapo ....EI^2O — From No. viii., $A^2I^2I^2$: Neg. 1.
19. Fresison........EIO — From No. xi., Neg. 1 : (=Ferio).

When *Fesapo*, as thus thrown into the first figure, is transferred from extension to comprehension, it gives $AE\frac{1}{2}E$, from No. viii.,

AA^2A, Neg. 2. The readings of the other moods of the fourth figure in comprehension, are supplied by the named moods of the first figure.

DIVISION III.—THE FUNCTIONS OF THE SYLLOGISM AND OF THE SYLLOGISTIC FIGURES.

Abbreviations of thought, and suppression of steps in reasoning.

96. Thought is more rapid than speech, by an excess which hardly can be over-estimated. Of the characteristics common to all languages, as well as of those which are peculiar to each, an immense proportion are, as it has justly been said, nothing else than expedients devised for the purpose of abbreviation. The inequality of the race causes diversity in the kinds of motion. Thought may proceed by steps; language, in the endeavour to keep up with it, must and does advance by leaps. When we throw our thinking into words, though it be only in self-communing, we continually pass, with a bound, over those steps which, for ourselves, are plainly and necessarily implied: when we aim at imparting our thoughts to others, we suppress similarly everything which it seems safe to leave in implication.

The desire of compression becomes especially active when concepts are the elements; for here we are forced on words, on common terms, as well in silent thought as in utterance. This is one of the causes why, although imagination may dart from object to object with the swiftness of the telegraphic flash, reasoning, burdened by its weight of words, must travel from truth to truth with comparative slowness. But we strive incessantly to expedite the journey.

Thus, as has already been observed, propositions, after having been distinctly thought, are made to yield complex terms, which supply their place: "This man is amiable," is substantially preserved in the form, "This amiable man." Such abbreviation gives, among other products, the means of substituting, for the cumbrous form of hypothetical reasoning, the simpler form of reasoning categorically. In the meantime, dealing with the latter species of thinking, we have to note the fact, that mediate inference brings the abbreviative tendency to light in another shape. In a large majority of actual instances, such inference assumes an appearance which disguises its real character. We have reached the utmost limits of direct abridgment: we strive towards the same end by having recourse to suppression. Not merely when we impart thought, but also very often when we think without communication, we evolve to ourselves, we extricate in explicit judgments and propositions, those steps only of our reasoning which the state of our knowledge causes to need explication.

We suppress one of the two premises of a syllogism: often we throw the whole of it into one complex assertion: "The X's are Y's, therefore they are Z's"; or, "The X's, being Ys, are Z's." A syllogism expressed thus incompletely is technically called an *Enthymeme*.[1]

The state of things may be best observed through the First Figure. This figure, when considered as to its matter,

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[1] This is the scholastic meaning of the word: but, as Hamilton has shown, it had, in its Aristotelic acceptation, no reference to form. Aristotle's *syllogism* was an inference in matter necessary; his *enthymeme* was an inference in matter probable. (See also Bachmann, p. 260.)

is a process in which a given principle or law, the compass of which is known, is applied to a given case, or cases. The major premise asserts that certain cases M are governed by the law P; the minor premise asserts that the given case S is one of those certain cases M; the conclusion asserts that the given case S is governed by the law P.

Now, in solitary thinking, it can happen but seldom,—in the transference of thought from mind to mind it will not happen very often,—that both of the antecedent truths shall demand to be explicitly brought up to the surface. In most instances, the premise which we wish to present clearly, to ourselves or to others, is the minor, asserting the identity of the given case S with one of the cases M. We follow this assertion immediately, by inferring that the case S is governed by the law P. The implied assertion of the major premise, that all the cases M are governed by the law P, may safely be left unexplicated if two conditions concur: first, that it shall be undeniable; secondly, that it shall necessarily be suggested by the two express assertions. The suppression of the minor premise may have place when our position is reversed. But that of the major is by far the more common: we leave the compass of the law unexpressed; we affirm only the inclusion of the given case in that compass.

The logical necessity of explicating suppressed premises.

97. The necessity of the suppressed premise, as one of the steps in mediate inference, is demonstrable indirectly, through the supposition of its being either supplied wrongly or denied.

Let the terms be symbols of indeterminate meaning: "All Y's are X's; therefore all Y's are Z's." Evidently there is here an argument, on the validity of which a person unfamiliar with logical analysis might be puzzled to decide

Every one would perceive that some assertion is implied: many might supply the deficiency wrongly. "All Z's are X's" might suggest itself to some, and would vitiate the argument. The right premise wanted, the major, is, "All X's are Z's." Let that premise be denied, and any one would pronounce the reasoning inconclusive.

When the terms are determinately significant, our previous knowledge, and our practice in reasoning, concur to make it very unlikely that we shall catch up a wrong premise instead of the missing one. But we shall always discover the necessity for supplying something. "This theory is a novelty in science; therefore it is dangerous." The required major is the assertion, that "All scientific novelties are dangerous." Those who admit this major must admit the conclusion: those who deny it are entitled to assert that the conclusion is unproved. "All Mr M.'s opinions deserve great deference: therefore the opinion that we can infer from one premise deserves great deference." Palpably, there is here implied a minor premise, asserting that the opinion inferred of is an opinion of Mr M.'s. It being admitted, the reasoning is good: if it be denied, the reasoning goes for nothing.

To the argument thus raised, no sufficient answer has been given, by any of those who have maintained the inessentiality of the suppressed premise. They have not been able to show how we could dispense with the *major* premise. Its suppression indicates, only, that the compass of the law which founds the argument is supposed to be so evident, and so directly suggested, as to make the statement of it unnecessary.[1]

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[1] There is felt, it must be confessed, a difficulty, seemingly insur-

98. With the suppression, much less usual, of the *minor* premise, the objectors have dealt less closely. But one argument referable to this premise brings out a distinction, which disposes of many very plausible instances urged as difficulties.

Supposed suppression of the minor syllogistic premise.

" A naturalist finds the remains of a horned quadruped, and pronounces that it was a ruminant animal. The reasoning here, if considered as class-reasoning, is perfectly expressed by a single premise with the conclusion. All horned quadrupeds are ruminant; therefore this horned

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mountable, in seeing how Mr Mill's acute and instructive theory of the syllogism affects the maior premise farther than by throwing it a step out of the way, while requiring also its being brought back, when required, for the thorough testing of the argument. " All inference is from particulars to particulars: general propositions are merely registers of such inferences already made, and short formulæ for making more. The major premise of a syllogism, consequently, is a formula of this description; and the conclusion is not an inference drawn *from* the formula, but an inference drawn *according to* the formula; the real logical antecedent or premises being the particular facts from which the general proposition was collected by induction. These facts, and the individual instances which supplied them, may have been forgotten; but a record remains, not indeed descriptive of the facts themselves, but showing how those cases may be distinguished, respecting which the facts, when known, were considered to warrant a given inference. According to the indications of this record we draw our conclusion, which is, to all intents and purposes, a conclusion from the forgotten facts. For this it is essential that we should read the record correctly; and the rules of the syllogism are a set of precautions to ensure our doing so." (*System of Logic*, book ii., chap. 3, sect. 4.) That the universal proposition, which is the major premise of the first figure, must, if it is a derivative truth (not otherwise), have been gained through antecedent induction, is a truth which cannot be too pressingly insisted on.

quadruped was ruminant. A logician may say, 'Yes; but you here comprise in the conclusion two facts or propositions; and when these are separated, you obtain a regular syllogism. All horned quadrupeds are ruminant; this quadruped had horns: therefore it was ruminant.' The introduction of a separate proposition, nevertheless, is obviously forced; it adds no strength to the inference, and represents no separate mental operation."[1]

The introduction of it would, at all events, be in many cases quite needless. Not a few of the stereotyped logical examples are in the same predicament with this, of seeking to expand into a syllogism an inference which may be regarded as not really syllogistic,—as being, not mediate, but really immediate. It has appeared already that a syllogism in the first figure is a process of double subalternation. When we can safely infer from the subalternant directly to a subalternate, we always do so; but, if we cannot, we may still be able to infer, from the subalternant, through a proximate subalternate, to the still lower subalternate in which we are interested. In the former case, we have an Immediate inference; in the latter, we have an inference Mediate or Syllogistic. In which of the two forms we shall either think or speak, is a question which we decide by considering the circumstances in which we are, or suppose ourselves to be. In the first form of the example quoted, the subalternate term, "this horned quadruped," expresses a complex idea, which is supposed to have been antecedently extricated from the proposition, "This animal is a horned qua-

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[1] Bailey, *Theory of Reasoning* (1851), page 81; a treatise whose objections to the received theory of the syllogism imply several very valuable suggestions.

druped." It is assumed, in short (as in an instance so simple it safely may be), that the fact of the animal being a horned quadruped is not worth explication into a proposition; and, on this assumption, our inference is immediate. But, if we were anxious to invite to that fact the attention either of ourselves or of others, we should require to explicate it: and the inference would become mediate. The minor premise would certainly, in this alternative, be expressed; and, further, if the compass of the law were doubtful (if, for example, we addressed ourselves to persons unfamiliar with zoology), the major premise would have to be expressed also.

In a word, arguments, where the minor premise seems to be suppressed, are really, in a majority of instances, as in this, cases of simple subalternation. There is no spell in the triplicity of the syllogism; and an inference not requiring expansion into the syllogistic shape, should never be violently stretched into it.

The function of the syllogism considered generally.

99. In every argument, then, which is actually thought as a mediate inference, two premises are necessary as the Antecedent; although, not in communication only, but also in uncommunicated thought, one of the two may be unevolved when the argument first presents itself, and may remain unevolved unless a call arises for analysis. The accusation made against the syllogism, of representing, as embraced in mediate inference, more steps than those which it really contains, cannot be entertained.

The alternative charge, that the conclusion of a syllogism is virtually implied in its two premises together, is true; and the admission will be estimated very lightly, by those who have a just insight into the close limitations which shut in

human reasoning on all sides.[1] Every original or primitive truth is individual, and is gained, not by reasoning, but by observation of our own thoughts or of the world around us. Even one such truth cannot be generalized, that is, it cannot be asserted to be a truth in more instances than one, unless through processes which are derivative thinking of one kind or another,—processes which must be inferences, either directly from one judgment to another, or indirectly from one judgment to another, through a third. The necessity of the truth revealed in a presentative cognition finds its normal expression in "this must." It is not till we have reflectively thought out the possibilities of generalization, that the same truth is expressible as necessary through "all are." Still more evidently is it impossible that, otherwise than through inference, such generalized truths can be brought to bear on cases, their application to which had not been directly observed.

Knowledge is digested through the two processes thus described: the ascent from this and that observed object to the generalized law of the class; the descent from the law to objects known only as included in the class. The processes are Induction and Deduction. Both are merely the disentanglement of relations given in complication, the distribution of known facts in masses as exponents of discovered laws. They yield systems in which our knowledge is symmetrically arranged, by in-

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[1] "The general principle of the syllogism is formal identity; that is, identity between the antecedent and the consequent. One of the premises must contain the conclusion: the other must declare that the conclusion is there contained." (Galluppi, *Lezioni di Logica e Metafisica,* ed. 1854, i. 306.)

duction according to the principle which rules its development into a whole, by deduction according to the principle which guides the determination of its several parts. That, by neither method of procedure, can any truth be discovered which is really different from the truths that lay at the root, is a fact which, while it springs necessarily from the limited character of human thought, does still leave to both methods their inestimable value. Between unreasoned knowledge, and knowledge systematically reasoned, there lies the world-wide distance between confusion and distinctness, between thick mist and brilliant sunshine, between the inert lifelessness of chaos and the rejoicing animation of the peopled earth.

On the operations which thus bring light out of darkness, logical laws merely keep a watch. They are guide-posts marking the track, topographical maps signalizing the points of the journey where thought is in danger of going astray. They are nothing more. Those laws of logical analysis, which require the throwing of the results of the operations into certain shapes, are only the alphabet through which we must read the inscriptions by the way-side, the key to the cypher which notes the facts discovered by the local survey.

This is the function discharged by all laws of Inference, from those of the simplest to those of the most complex kinds. It is emphatically the function of the laws ruling the Categorical Syllogism, a process which is the central point of all derivative thinking, rising above and passing beyond immediate inference on the one side, and standing on the other as the basis of all those more complex reasonings which take the more difficult syllogistic forms.

The special functions of the first figure.

100. The specific uses of the Syllogism vary with the several figures. Enough has been seen already to show that none of the first three, which only deserve scientific recognition, can be without applicability. We gain a prospect of the superficial relations between the syllogistic figures, in the course of that coasting voyage which we pursue under the pilotage of the scholastic rules; but the system of stratification, which contains the wealth of the gold-region, lies concealed, until, having mastered the doctrine of the Wholes of Predication, we travel into the heart of the country, to survey it as mining engineers.

The First Figure is the characteristic expression of knowledge already systematized, of *deduction* from principles accepted as ruling within a certain sphere.

When, for the explication of such knowledge, mediate inference is requisite, it is almost always, if not without any exception, because of an occasion presenting some fact or facts, whose subjection or non-subjection to a known law is not immediately obvious. We know the law; and we know its compass: there is thus matter for a major premise: all facts of a certain kind are either covered by the law, or are beyond the sphere of its operation. We know, likewise, that the fact about which we wish to reason is one of the facts which thus stand within or without the domain of the law; and this knowledge supplies a minor premise. Our data having thus been placed in exact relations to each other, there follows, inevitably, the judgment, that the narrower fact is subject or not subject to the law. The inference has been set in a form making it both clear and readily testable: and this formal setting forth of it has been made possible, by our having already arranged the three terms in a scale of ordination.

Perfect as a form of inference, the first figure is, just because of the regular sequence of gradation which it assumes, less likely to occur in ordinary thinking, whether with or without expression of both premises, than either of the other figures. In many actual cases, if not in most, the consequent is attainable through an immediate subalternation. Its uses are scientific oftener than popular. But it is invaluable as exhibiting the principle on which mediate inference must ultimately rest; and as thus being, directly or indirectly, the most decisive instrument for testing the validity of arguments that are either disputed or not distinctly wrought out.[1]

The special functions of the second figure.

101. The Second Figure expresses a knowledge deficient

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[1] To the first figure is applicable, one might even say exclusively, Mr Mill's description of the function of the syllogism (note to section 97), as being a code of rules for the interpretation of that abbreviated record of knowledge, which is embodied in universal propositions.

The protest of Ramus, against the speculative tendencies of the Aristotelians, guided him and his followers to some instructive views as to the functions of the several syllogistic figures. They declared the third figure to be (oftenest in an enthymematic form), the first and most natural mode of dianoetic or discursive thinking. Dividing, as usual, by dichotomy, they placed over against it a class containing the other two figures. But, in that class, the second figure, as having the simpler formal relation between the middle and the extremes, stood before the first. "The figure which Aristotle calls the first, is in the order of nature the last." This remark is Milton's, whose logical treatise illustrates very ingeniously the Ramist system of dialectics. (Compare Ramus, *Institutiones Dialecticæ*, lib. ii., capp. 10, 11, 12; with Milton, *Artis Logicæ Plenior Institutio*, in his Prose Works by Birch, ii. 545–551.)

by one step only, but that a step so important as seriously to cripple the inference.

The thinking which it expresses is indistinct, in the wider of the two judgments which supply the premises. In the *major* premise, as it has been shown, we turn aside from the route of deduction. We do not directly think the compass of the law, either positively or negatively: we do not explicitly place our intermediate cases either within the law or out of it. We start from the thought of the law itself, and predicate of it that it lies out of all the intermediate cases. Doubtless, this assertion implies that the cases are out of the operation of the law; but it does not clearly express the thought of this second assertion. The difference of form or expression is symptomatic of a real difference in thinking: the source of the difference lies in our not having systematically ordinated the thoughts denoted by the three terms; and we suffer for the shortcoming, by being tied down to a negative major premise. The inclusion of our given narrowest case among the intermediate cases, yields a minor premise; but the exclusion of that case from the law is the only consequent attainable.

If, however, a negative conclusion only is aimed at, this figure is equally available with the first; and, when the major is the suppressed premise, it will be filled up for either figure, according to the greater or less distinctness with which the thinker has classified his knowledge of the matter handled. Where, indeed, the aim is the detection of differences, while positive attributes, as clearly known, are not attended to, our familiar deductions are likely to fall into the second figure rather than into the first. For the law of identity and non-identity, which glimmers out from afar, above all our thinking, as the twin star by which

it must always steer, has its most obvious bearing when the middle term has the same function in both premises.

The consideration last hinted at is applicable with peculiar force to the remaining figure, which, as having distinctive uses infinitely wider than the second, must receive much closer attention.

102. The practical uses of the Third Figure are both more various, and more firmly marked, than those served by either of the others.

The special functions of the third figure.

It is distinctively the *exceptive* figure. A law being asserted as universal, the exhibition of any instance (our middle term) in which it is violated, entitles us to deny the universality in our conclusion. Both positively and negatively, also, it is the form by far most natural for *exemplification:* the middle term is set forth as being, in a given class of objects, an instance in which a law is either obeyed or not obeyed; and hence we infer that there are instances in which the law either holds or does not. Further, both exception and example are sufficient, though there be but one instance of the sort. Hence our middle term is often a singular. This figure lends itself easily to the reception of such a middle, while no other will: and even uninstructed thought, guided by a twinkling suspicion of the ultimate laws of thinking, throws such reasonings into a shape in which the third figure is involved. In all applications such as those just described, the argument proceeds safely, and needs little or nothing either of warning or of guidance.

But the fact stands differently in regard to the most important of all the uses to which the figure may be applied; a use, indeed, towards which the others are only the first steps.

The third is distinctively the *Inductive Figure:* and its character, as applicable to this purpose, must be looked at with all the closeness which our opportunities permit.

When we reason in the third figure, we start, as in the second, from a knowledge which is, at one point, incompletely systematized. But now the cloud overhangs the opposite quarter of our horizon.

We possess the law; and we know its compass, either positively or negatively. The major premise asserts, of its two terms, the relation which they would be found to bear if our terms were thought in their just ordination: our intermediate class of cases is governed by the law, or disobeys it. It is in the *minor* premise that the clew of the deductive maze has been lost. We cannot there assert, that the case or cases as to which we desire to infer, or any of them, are included in the intermediate class; we can assert only that the intermediate class, or some part of it, is included among the cases about which we are directly concerned.

Our position is seductively promising. The intermediate class, denoted by our middle term, is pronounced to be, wholly or partly, either identical with both of the classes denoted by our other terms, or identical with the one of the two, and non-identical with the other. But, in the step signified by our minor premise, we have turned aside from the deductive sequence; and the penalty must be paid. Our conclusion is valid only as to a part of the class of cases about which we aimed at inference. We are not secured against disappointment unless, being forewarned of the limitation, we have in the beginning narrowed our sphere to a part of that class. If, not having thus protected ourselves, we draw an universal conclusion, we have stumbled into an illicit process of the minor term.

The process which has thus been described, from the objective side, is that which bears the name of Induction. Our universal affirmative propositions, those which express the whole compass of laws, and which become available as the major and confining premises for processes of deduction, have, if they are truths derived from others, been antecedently gained, by us or for us, through induction. Further, those inductions, as actually performed, proceed from data no wider than those explained here, and in the formal scrutiny of the third figure. Yet induction takes place, naturally and usually, in forms which, when completely and exactly set forth, fall into the third figure: while, if it be, in certain circumstances, referable directly to the first, its mood is inevitably one of the two which authorize only particular conclusions. Consequently, an incalculably large proportion of the universal affirmatives, from which, in deduction, we travel downwards, have been reached by a method whose prohibitory laws have been disobeyed. Our ordinary inductions, having conclusions universal instead of particular, are logically inconclusive. Some logicians have rightly called them Imperfect Inductions. There is, indeed, a possible process, describable as a Perfect Induction. As deduction is valid only from a whole to any of its parts, from a genus to any of its species; so induction is valid or perfect only from all the parts to the whole, from all the species to the genus. If, being able to assume only that a law governs some of the species, we hence infer that it governs the genus, our induction is imperfect, and our inference fallacious. If, being able to assume that a law governs all the species, we were hence to infer that it governs the genus, our induction would be perfect, and our inference valid. But the data for such a process are

never extant when they are most wanted: our common procedure does never, in the most favourable circumstances, supply them completely. All these truths must be resolutely faced.

Syllogisms expressing a Perfect Induction could not fall within any form embraced in the common scheme. Their moods, in affirmation and negation severally, would be AA^2A and EA^2E. The minor premise in each of these does, in fact, assert a Logical Division: and therefore it assumes, as given, a knowledge of all the species by which a genus is constituted. But, when this is our position, we have already, in effect, generalized to the utmost extent which our data allow: specification, through deduction, is the only new process we can have a real interest in undertaking. Perhaps no man ever found it worth while to infer, explicitly, that, because something is true of all the contained species, it is true likewise of the containing genus. In a word, the perfect induction, as being the counterpart of the only valid deduction, is speculatively important and interesting; but the practical functions which the syllogism discharges, when it is used as an instrument of generalization, must be appreciated through those other inductions, which are imperfect and therefore formally invalid.[1]

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[1] The Perfect Induction is dealt with by Joannes Major, in one of the passages already quoted from (note 2 to section 39). Derodon exemplifies it by the following syllogism, which is really in AA^2A of the third figure: "Ignis, aer, aqua, et terra, sunt corpora; sed ignis, aer, aqua, et terra, sunt omne elementum: ergo omne elementum est corpus." (*Logica Restituta*, 1659, p. 602.)

The following are the two formulæ of perfect induction proposed by Sir William Hamilton in 1833:—

103. The Third Figure, while it may, doubtless, like the second, be adopted needlessly, is yet the only form which inference can naturally assume when we are bent, not on determining or specificating universal truths already known, but on enlarging our knowledge, by widening our sphere of generalization. In deduction we argue from the subalternant to the subalternate, from a given class to something contained in it. "Because all are, some are:" the inference is good. In the imperfect induction we argue from the subalternate to the subalternant, from something given as contained in a class to the class itself. Our inference is bad, as passing beyond the sphere of our immediate premises: whether we conclude, peremptorily, that, because some are, all are; or only that, because some known objects of the class are, therefore some unexamined objects of it must be also.

The bearings of the third figure on the imperfect induction.

It is because of the logical weakness, which thus pervades all ordinary inductions, that arguments from experience, analogy, or example, in questions relating to human character and conduct, are so apt to be delusive, and require to be scrutinized with so much jealousy. It is because of the same weakness, that arguments of the same type, when used as instruments in the construction of

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X, Y, Z, are A :		A contains X, Y, Z.
X, Y, Z. are (whole) B :	or,	X, Y, Z, constitute B.
∴ B is A.		∴ A contains B.

See his *Discussions*, p. 161 ; and compare Baynes, *New Analytic*, pp. 71, &c. Consult also Mansel, *Prolegomena Logica*, pp. 207–211; Trendelenburg, *Logische Untersuchungen*, ii., pp. 261–3 ; Drobisch, *Neue Darstellung*, §§ 140–146. On the objective side, see Whately, *Elements of Logic*, book iv., chap. i., § 1.

scientific systems, are felt to need fencing round by an array of checks and counter-checks. Such an array constitutes the code of laws which is usually called the Philosophy of Induction; a code diversely promulgated by diverse lawgivers, and admittedly susceptible, in all its editions, both of improvement and of enlargement.

Perhaps the simplest view which can be taken, of the design aimed at in the inductive laws, is this. The imperfect induction leads to a conclusion, which, if stated as universal, involves a logical error: it is required to reduce that error to as narrow a limit as possible; and it is an end not only desirable, but in many cases attainable, that the error shall be narrowed to a minimum which is practically inappreciable. If, the conclusion being stated as an universal ("all are" or "all are not"), the attempt is yet made to give warning of its amount, that amount might be indicated, in the predicate, through the degrees of a modal scale, loosely indicable thus: "Perhaps, probably, very probably, probably in a very high degree, probably in the highest degree thinkable below demonstrative certainty." If, on the other hand, it were attempted to intimate the amount of the error through the subject of the conclusion, the notice might be given through a corresponding scale of quantitative symbols: as, "A few, many, very many, almost all, all that there is any reason for believing to exist." The raising of the inductive conclusion to the highest of these degrees, whether of qualitative probability, or of quantitative inclusion, may be said to be the end aimed at through the inductive laws; such laws, for instance, as those which Mr Mill proposes under his four "Methods of Experimental Inquiry," the methods of Agreement, Difference, Residues, and Concomitant Variations.

How are any such laws effectual? And how is truth attainable through a process which, so far as we have yet examined it, appears to leave open the chance of error?

Laws are applicable to the case, truth is attainable through the process, by reason of this fact. The explicative process is and must be, at one stage or another, ampliated by an assumption not implied in the data. Further, that which is assumed must be a necessary and universal truth; it must be a truth which thus lies above the universe of experience, but which becomes known to us only through our experience of individual facts, while it is expressible only by reference to the relations under which those facts became objects of cognition. When regarded in its most concrete and complex aspect, the principle is spoken of by such phrases as "The uniformity of nature." Specificated from a higher point of view, it yields the assertion that "Every fact or phenomenon is governed by a law or laws:" and, when we look down on it from a station yet more elevated, it resolves itself into the doctrine, that "Like causes produce like effects."

This principle rules human activity quite as sternly as it rules the passivity of corporeal matter. But the laws, the causes, are not the same. Will, indeed, predominates over both regions; that all-directing and sustaining Will, in the thought of which is found the last solution of the problem raised, when we trace law and cause upwards into pre-formed purpose or design. That universal energy, however, being reverently taken for granted, we see that mind exercises volition of its own, that it acts, or exercises its powers, in virtue of will; while body merely obeys the universal laws of its nature, whether left to itself, or influenced, in conformity with these, by the will of man. Hence it is that the

generalization of reasonings about mind is, and must always continue to be, so much more hesitating and precarious than similar generalization about things corporeal. Given a cause, that is, given all the elements of a cause; it must always be possible to prognosticate the effect. But we may know all the causes which immediately influence body: we never do or can know all the causes which immediately influence mind; and this impotence imposes a limit on the certainty, both of a generalized law, and of its application to an individual fact.

The uses of syllogistic reduction.

104. Though the fourth figure is certainly useless, it is clear that both the second and the third represent forms of reasoning, which are not only actual but frequent; while the third, likewise, is the natural form of our common generalizations.

Reduction of syllogisms, then, to the first figure, cannot well be maintained to be necessary on a ground which has been hinted at by one great philosopher, and regularly developed by another.[1] The imperfect moods, it has been said, are really Mixed Inferences: each of them implies one or more immediate inferences by conversion: and it is

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[1] The hint is Aristotle's: *Analytica Priora*, lib. i., cap. 1; *sub finem*. It is developed by Kant, in one of his minor treatises: *Die falsche Spitzfindigkeit der Figuren*. By him, and those who have followed him most closely in the application of his doctrines to logic, such as Kiesewetter, the indirect syllogisms are called Mixed or Hybrid. Schulze calls them syllogisms Extraordinary or Transposed: *Grundsätze der allgemeinen Logik*, ed. 1831, p. 118. Kant's view had been virtually anticipated by the paradoxically acute Derodon: *Logica Restituta*, p. 647.

from these implied premises, not from the expressed ones, that the conclusion is mediately inferred. The substituting of the implied propositions for the expressed ones, is the reduction of the syllogism.

The process of thought supposed by this theory, is not that which actually takes place. When an argument is expressible as a syllogism in a mood of any indirect figure, we have really thought in that figure, not in the first.

But reduction, though it does not show what the given inference really was, does show what it might have been: and herein is its usefulness to be found. We had fallen into the given figure, because, whether through imperfect knowledge or through want of reflection, we had not distinctly thought the three concepts of our reasoning, in the relations which they must bear to each other as members in a classified series. Reduction brings those relations to light, exhibiting the three terms as being, successively, contained, containing and contained, and containing. Now, likewise, there becomes directly applicable to our argument the law of subalternation, the highest concrete principle by which reasoning through common terms admits of being tested.

Accordingly, the transformation of Indirect syllogisms into Direct, should be considered as being only a valuable means of analysing, to the furthest possible point, the elements of a given argument. Every indirect syllogism may, with immediate reference to its conclusion, be held to be a genuine form of thought, resting on a foundation which, though neither the widest nor the deepest of those that underlie it, is yet perfectly strong enough for its support.

Now, therefore, when our study of the simple categori-

cal syllogism is completed, not only may we ask, whether the attempt has been made to propound any laws governing, indifferently, syllogisms of every form; but we may also collect some of those many codes of maxims, in which it has been attempted to draw to a point the functions and formal rules of the several figures.

Specimens of proposed syllogistic canons.

105. The question as to universal laws of the syllogism reminds us of the two Syllogistic Canons. These, the oldest laws of the sort, were sufficiently examined in a preceding division of the present chapter. But emphatic commemoration is here deserved by that resolution of the canons into the laws of identity and difference, which was then quoted from Smiglecius.[1] Somewhat later than his time, the same reference was carried yet farther. "For all syllogisms," says Derodon, "whether affirmative or negative, this one principle is sufficient; that things which are the same with a third thing are the same with each other. The principle requires no limitation whatever."[2]

In the logical systems of the modern Germans, the ultimate dependence of the syllogistic rules on the laws of identity and difference, is generally insisted on, and more or

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[1] Section 82; and its second note.

[2] *Logica Restituta*, pp. 642, 644. Negative syllogisms he brings under the law of affirmation by contraposition. The absolute identity of the objects denoted by the extremes he maintains chiefly on metaphysical grounds, similar to those urged by Smiglecius; but one section in his argument shows him to have apprehended (not very distinctly) the doctrine, that the quantitative signs are integral parts of the terms.

less satisfactorily traced. But we do not encounter, among them, more than a solitary attempt to deduce, for the syllogism, one universal canon, exhibiting the application of the two axioms to the concept. These logicians, however, are very lavish in generalizations of the rules and characteristics of the several figures; and some of the doctrines they have proposed may usefully be cited.

In the First Figure, says Lambert, the middle term is a ground or reason; in the Second it is a difference; in the Third it is an example; in the Fourth it is a ground of reciprocity. "I. The law of the First Figure is the *Dictum de omni et nullo:* What is predicated of all A's, may be predicated of every A. II. The law of the Second Figure is the *Dictum de diverso:* Things which are different, cannot be predicated of each other. III. The law of the Third Figure is the *Dictum de exemplo:* When we find things A which are B's, there are A's which are B's. IV. The law of the Fourth Figure is the *Dictum de reciproco.* (1.) If no M is B, no B is this or that M. (2.) If C is or is not this or that B, there are B's which are or are not C's."

By the same close analyst, the functions of the four figures are otherwise distinguished in this way:—"(1.) The First Figure appropriates to the thing what we know of its attribute. It infers from the genus to the species. (2.) The Second Figure leads to the difference of things, and removes confusion of concepts. (3.) The Third Figure gives examples and exceptions for propositions which appear universal. (4.) The Fourth Figure finds species for the genus, in *Bramantip* and *Dimaris;* it shows that the species does not exhaust the genus, in *Fesapo* and *Fresiso;* and it denies the species of that, whereof the genus is denied, in

Camenes."[1] We shall not again find mention of the fourth figure.

Consideration is called for by a generalization wider than this. Herbart distributes the three figures into two classes. The First and Second are called Syllogisms of Subsumption, in respect that in each of them the minor term is subsumed or subordinated under the middle: the Third Figure is said to have Syllogisms of Substitution, their character consisting, it is alleged, in the substitution of one term for another.

One of the most acute thinkers of Herbart's philosophical school, while dissenting in part from the distribution, has expressed the laws of the three figures with pregnant brevity. "The First Figure," says Drobisch, "may be said to reach its conclusion through subsumption, the Second through opposition, the Third through substitution."[2] He assigns both a universal law of the syllogism and special laws for the three figures, in the following propositions. In

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[1] Lambert, *Neues Organon*, 1764, vol. i., pp. 136–143; "Dianoiologie," section iv. Mr Thomson has traced the substance of Lambert's laws, except that for the fourth figure, to Keckermann; and perhaps they are still older. (Thomson, *Laws of Thought*, p. 228; Keckermann, *Systema Logicæ* [*Plenius*], ed. 1614, lib. iii., capp. 5, 6, 8, 9, pp. 746, 756, 757.)

[2] Herbart, *Einleitung in die Philosophie*, ed. 1850, p. 111. Drobisch, *Neue Darstellung der Logik*, ed. 1851, sections 81–88. The distinction between Darapti and Felapton on the one side, Datisi and Feriso on the other, was not overlooked by Herbart. But, followed by Drobisch, he uses it by founding on the double distribution of the middle in the first two of those moods, and taking them as the norms of his "substitutive inferences." Surely, however, the character of the third figure is very loosely apprehended when it is said to rest on substitution.

the first of these, M denotes the middle term, A and B the terms of the conclusion; the functions of all these terms, as subject or predicate, being left unfixed.

"A conclusion will always be yielded, when it can be shown, that the whole or a part of the sphere of A, through its relation to the sphere of M, and the relation of M to the sphere of B, is either contained in the sphere of B, or excluded from it.—I. For obtaining syllogisms of the first figure in conformity with this universal principle, the application of the two following laws is sufficient: (1.) In that, wherein the whole is contained, there is contained also its part; (2.) From that, wherefrom the whole is excluded, there is excluded also its part.—II. Syllogisms in the second figure are yielded through application of these two laws: (1.) The part of a whole is excluded from that which is excluded from the whole; (2.) That which is excluded from the whole is also excluded from its part.—III. Syllogisms in the third figure are yielded through application of this law: Identical determinations of a concept may be substituted for each other."

Twesten gives thus the laws of the three figures. "I. In the First Figure, we infer from the genus to that which is under it, by the so-called *Dictum de omni et nullo.* It may be regarded as a widened subalternation. II. In the Second Figure, from the opposite relation of two concepts to a third, we infer to their own opposition. It may be regarded as a widened opposition. III. We may consider the Third Figure as an application of the analytic [explicative] law, that there is given, with a concept, the agreement of its marks. The procedure in it is according to this principle: Concepts which may be predicated of the same subject, may be predicated of each other, though only with

limited quantity or modality; concepts, on the contrary, of which the one, but not the other, may be predicated of a certain subject, may be denied of each other, under the same limitation as before."[1]

Sir William Hamilton's syllogistic canons.

106. The only other shape of the laws calling for citation, is that in which they have most recently appeared, and in which they are designed to cover, not only the current syllogistic scheme, but likewise all the new moods proposed by their author.

The form in which Sir William Hamilton expresses his one universal canon of the syllogism is this. "What worse relation of subject and predicate subsists between either of two terms and a common third term, with which both are related, and one at least positively so: that relation subsists between those two terms themselves." The same law is given by Mr Thomson, in a form bringing it nearer to being a combination of the two received canons. "The agreement or disagreement of one conception with another, is ascertained by a third conception; inasmuch as this, wholly or by the same part, agrees with both, or with only one, of the conceptions compared."

To the work last quoted from, the author of the universal canon furnished also the following specificated applications of it to the several figures:—

"*Figure I.*—In as far as two notions are related, either both positively, or the one positively and the other negatively, to a third notion, to which the one is subject and the other

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[1] Twesten, *Die Logik, insbesondere die Analytik,* 1825, §§ 105–109.

predicate; they are related, positively or negatively, to each other as subject and predicate.

Figure II.—In as far as two notions, both subjects, are, either each positively, or the one positively and the other negatively, related to a common predicate-notion; in so far are those notions, positively or negatively, subject and predicate of each other.

Figure III.—In as far as two notions, both predicates, are, either each positively, or the one positively and the other negatively, related to a common subject-notion; in so far are those notions, positively or negatively, subject and predicate of each other."[1]

The propositions which have been cited, in this section and the last, may suggest more reflections than one. Each of the codes of laws struggles towards one principle, and is clearly intelligible when that principle is understood and remembered: we are led, everywhere, to look back on the law of non-contradiction, with a beckoning, more or less emphatic, towards the character of concepts as the objects on which the law is brought to bear. Each of the codes, again, is, for those who have mastered the received rules of the syllogism, easily interpretable, as being nothing else than a generaliza-

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[1] Baynes, *New Analytic*, p. 53. Thomson, *Outline of the Laws of Thought*, pp. 214–230. We have it not now to learn that particularity is a "worse" relation than universality, negation than affirmation. For the full development of Sir W. Hamilton's scheme, however, it has to be remembered, that in extension the subject is "worse" than the predicate, being thought as quantitatively a part of it; that in comprehension, for the same reason, the predicate is "worse" than the subject.

tion of them. But, on the other hand, the bolder the generalization is, the more difficult does it become to descend again to the received rules; if, indeed, we have not, at one or two points, been guided completely beyond sight of them.[1]

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[1] The following works may be referred to, for other comparisons of the figures:—Melanchthon, *De Dialectica*, lib. iii.; *La Logique* (de Port-Royal), *partie iii.*, chaps. 5, 6, 7; Wolf, *Philosophia Rationalis*, 1728, pp. 311, 317, 320; Wyttenbach, *Præcepta Philosophiæ Logicæ*, part iii., chap. 6, § 13; Maass, *Grundriss der Logik*, ed. 1806, p. 222—(the only one of the recent German logicians, so far as we know, that has attempted to generalize the rules of the fourth figure); Hoffbauer, *Anfangsgründe der Logik*, ed. 1810, pp. 164, &c.; Kiesewetter, *Grundriss einer allgemeinen Logik*, ed. 1824, vol. i., pp. (109), 403, 405, &c.; Fries, *System der Logik*, ed. 1837, p. 165; Kidd, *Primary Principles of Reasoning*, 1856, chap. v., sect. 4. For Mr Mill's reading of the law of the first figure, see his *System of Logic*, book ii., chap. 2., §§ 3, 4; and Mr Kidd's observations on the passage in chap. iii., § 1.

CHAPTER IV.

COMPLEX MODES OF INFERENCE.

DIVISION I.—INFERENCE BY COMBINATION OF CATEGORICAL WITH NON-CATEGORICAL PREMISES.

The character of conjunctive propositions.

107. We assert and reason categorically, through assumption of the relations between terms. This is the form which judgment naturally and spontaneously takes when its data are positively assumed; and no long series of judgments deviates steadily from it. That we may be able to adopt it, we continually throw into the shape of complex ideas and terms the judgments which we desire to develope further. To this form, likewise, thought must be reduced, before the primary logical laws can be made to bear directly on it.

But there are propositional forms which are not categorical. Such may be said to be all those with which, under the name of "Exponible" or "Compound" prepositions, we have already made up a passing acquaintance.[1] These are formally distinguished from categoricals by their complexity: each of them is constituted by two or more propositions that are categorical. Each of them, again, may be analyzed into its constitutive categoricals. Further, most of the kinds of exponibles do not become rightly avail-

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[1] Note II. to Part II., Chapter I.: *Interpretation of Propositions.*

able as data for inference, till that analysis has been performed. The inferences then issuing from such propositions are traceable immediately to the relations of the terms, and consequently are ruled directly by the laws of categoricals.

This limitation of use, however, does not affect all exponibles. The exception which has place is indicated by a difference in form. While, in most kinds, certain of the constitutive propositions are only implied, there are two of the kinds in which all the constitutive propositions are explicitly set forth. Propositions of those two kinds may be inferred from, without being subjected to an analysis deep enough to lay bare the implied relations of the terms. In inferring from them, we may content ourselves with assuming a relation, explicated in our complex proposition, between the propositions which constitute it. The peculiar character of the relation so assumed, impresses a peculiar form on the inference for which it becomes a datum.

The two kinds of complex propositions, which are susceptible of being thus dealt with, may have their common character indicated if they are called Conjunctives.

A Conjunctive Proposition neither affirms nor denies any of the constitutive propositions: it merely asserts a relation between them. It is either an Hypothetical (or Conditional) proposition, or a Disjunctive.[1]

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[1] The name Conjunctive, for the genus, is that of the Port-Royal Logic, as well as of earlier works. There is an awkwardness, obvious and undeniable, in adopting this as a generic name, while the name disjunctive is applied to one of the two species. But the fault seems to be less than that of the terminology adopted by Whately, and others, from Aldrich, Sanderson, and most of the

An Hypothetical proposition is constituted by two categoricals; and it asserts that, on the hypothesis or condition that one of the constitutive propositions is true, the other is true also. "If X is Y, (then) Y is Z:" or, "If X is Y,

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older English logicians. These give to the genus the name "hypothetical;" and they designate the two species as "conditional" and "disjunctive." But the words "hypothetical" and "conditional" are palpably synonymes: nor is the name "hypothetical" very apt for disjunctives.

In their treatment of the complex modes of inference, the German logicians, almost to a man, are elaborately and most ingeniously minute; but, alike in nomenclature, in method, and in resulting theory, they are as discordant as the extreme difficulty of the problems would make one expect to find them. Most of them cling, more or less closely, to the remarkably subtle analysis of Lambert. From him, likewise, one or two English writers have taken some of the mnemonic names ("Saccapa, Caspida," &c.), by which, emulating the "Barbara" verses, he sought to symbolize the rules of those complexly-complex inferences, a few of which are touched on in the third division of the present chapter. It is but a very small part of those speculations that can be put to use in the summary here attempted.

Fries's distribution of judgments and propositions (*System*, p. 102), is worth notice for its bearing on complex inferences. Judgments fall, in respect of *relation*, into three classes. The relation of subject and predicate gives the Categorical Judgment; the relation of reason and consequent gives the Hypothetical; the relation of co-ordinates to the containing whole gives the Divisive. The divisives, again, are of two species: the Conjunctive, which is the "copulative" of many old logicians; the Disjunctive, which is the proposition commonly bearing that name. Copulatives, however, as was already observed, are really categoricals (Note II. to Part II., chap. i.); and, besides, they give but imperfect expression to that divisive relation, which is adequately set forth by disjunc-

(then) Y is not Z." The conditioning proposition is called the Antecedent, the proposition conditioned is the Consequent. The two propositions, accordingly, have functions corresponding, severally, to those discharged by the two terms of a categorical. A function parallel to that of the copula belongs to the "Consequence;" that is, the words "if" and "then," which express the relation between the constitutive propositions. Through these words (or more commonly, in our language, through the former alone), there is denoted the affirmation of the consequent, on condition of the affirmation of the antecedent.

A Disjunctive proposition is constituted by two or more categoricals: it asserts that one or another of these must be true, and the others false. "Either B is X, or B is Y, or B is Z;" or, "Either B is X, or C is X, or D is X." In almost every actual case, one of the terms (simple or complex) is, as in these examples, common to all the constitutive propositions; and, when the fact is so, the disjunctive is conveniently abridged into a form which would make it useable as a categorical having one term alternatively complex. "B is either X, or Y, or Z: Either B, or C, or D, is X." The place of the categorical copula is taken by the "Alternative" words "either" and "or." The import of these is double: they denote the affirmation of one or another of the constitutive propositions, and the denial of all the others. The constitutive propositions, again, represent the categorical terms; but none of them is tied down, as in hypotheticals, to the function of antecedent or conse-

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tives (compare section 56). The same remarks are applicable to the propositions which Drobisch calls "Divisive," as "B is partly X, partly Y, partly Z."

quent. For the character of the disjunctive relation involves an inconsistency, absolutely reciprocal, between any one of them singly, and each and all of the others.[1]

108. When conjunctive propositions are considered as premises or antecedents of inference, three points come to light.

Conjunctive propositions as antecedents of inference.

First: The only inferences affected by the conjunctive character are mediate. The conjunctive proposition is one premise: a second proposition must be supplied as the other.

Secondly: A conjunctive proposition of either kind may, with another premise of the same kind, yield a conclusion also of the same kind. But such syllogisms, purely hypothetical or disjunctive, neither evolve, nor depend on, the assumed relations of the constitutive propositions: they are genuine categorical syllogisms, hidden under a disguise, which is thrown over them by the uncertainty of the thinker as to the legitimacy of the assumptions they postulate. The pure hypothetical inference springs from doubt as to the truth of the premises: the pure disjunctive inference

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[1] Propositions disjunctive by negation are such as the following; "B is neither X, nor Y, nor Z: Neither B, nor C, nor D, is X." Evidently, however, these fall short, by more features than one, of the character assigned in the text to the disjunctive proper. Indeed, they may rightly be treated as affirmative categoricals, having one term negatively complex: "B—is—something which is neither X, nor Y, nor Z: That which is neither B, nor C, nor D—is—X." The introduction of such a proposition as a premise, when the other premise is categorical, would leave the syllogism directly amenable to the categorical laws.

springs from doubt as to the extension or comprehension of the terms.

The only syllogisms which do depend on and evolve the relations of the constitutive propositions, are those mixed ones in which the conclusion is categorical. The major premise is, for both kinds, the given conjunctive proposition. But, for both kinds, the minor premise must be categorical. In other words, the conditional or disjunctive relation is evolved, by being brought to bear on an unconditional and positive assertion of fact. In mixed hypothetical syllogisms, the minor premise is categorical, both formally and in substance. In mixed disjunctive syllogisms, the minor premise has often the disjunctive form; but it is always in substance categorical, an unqualified assertion of identity or difference between its subject and its predicate.

Thirdly: There appears a point, which, for the theory of these inferences, is the most important of all. The validity of categorical inference, from premises one of which is conjunctive, rests on this postulate: that the conjunctive premise shall be accepted as an affirmation of the result of certain antecedent processes. These processes have, for the two kinds, different characters, and rest on different principles.

(1.) An hypothetical proposition affirms the validity of an antecedent inference. It asserts that, the antecedent being admitted, the consequent must be inferred from it. The necessity of the inference may appear on the face of the hypothetical proposition; but much more frequently it does not. In either case, the proposition merely asserts that the inference is valid, while it presupposes the process by which the validity is established. It thus depends on one or more

of the laws of inference, and is traceable through them to the axioms of identity and difference.[1]

(2.) A disjunctive proposition affirms the completeness and accuracy of an antecedent process, which either is a logical division, or is ruled by the same principles as it. It asserts that the constitutive propositions set forth all the dividing members of the whole, and that these members exclude each other. It thus depends on the axiom of excluded middle.

This analysis of disjunctives covers all the actual cases. First: None of the constitutive propositions may have any of their terms identical: "Either B is X, or C is Y, or D is Z." Such a proposition is easily referable to a division having a very wide divisum: "All possible cases are cases, either of B being X, or of C being Y, or of D being Z." Secondly: Each of the constitutive propositions repeating one term, the disjunctive proposition may have singular terms only, and will thus assert, alternatively, individual identities only. In this unusual case, the assertion depends, obviously as well as directly, on the law of excluded middle. Thirdly: When its terms are common, one of them occurring in each of the constitutive propositions, the same dependence holds through the law of the concept. In such cases, a disjunctive proposition, in its abridged form, really asserts, of one of its terms (usually, but not necessarily, the subject),

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[1] Kant, and many other foreign logicians, place the hypothetical proposition by itself, alleging that it requires, for its justification, the doctrine of the "sufficient reason." But it was maintained, in a preceding note (section 15), that this doctrine, if understood as the assertion of a law purely formal, is resoluble into the axioms of identity and difference.

that its extension is constituted by the terms constituting the predicate. The predicate is an enumeration of co-ordinate terms, which are parts, and all the parts, of the extension of the subject. The proposition (if held to be a categorical, as it may be), is an A^2; and it affirms, or implies, a logical division carried down to one step only. It is so expressed as to signify, both the mutual exclusion of the parts, and the equivalence of their sum to the whole. Thus, the assertion that "All the B's are either X's, or Y's, or Z's," is interpretable as, or out of, this assertion; that the class B is constituted in extension by the three sub-classes X, Y, and Z. Categorical inference from such a premise is not possible, unless the members of the division are both mutually exclusive, and in their combination exhaustive.

The structure and rules of the hypothetical syllogism.

109. From the description of the hypothetical proposition, there come at once the two laws of the Categorico-Hypothetical Syllogism, which is oftenest described simply as Hypothetical. Each governs one of its two moods,—the Constructive or Positive mood, and the Destructive or Amotive.[1]

(1.) The Mood of Position rests directly on the principle of inference: if the antecedent is admitted, the consequent may be inferred. The minor premise, therefore, is an affirmation of the antecedent, the conclusion an affirmation of

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[1] Mood of Position, the "*Modus ponens*, a positione antecedentis ad positionem consequentis:" Mood of Amotion, the "*Modus tollens*, ab amotione, remotione, vel eversione consequentis ad amotionem antecedentis."

the consequent. "If X is Y, Y is Z; but X is Y: therefore Y is Z." "If every X is Y, no X is Z; but every X is Y: therefore no X is Z."

(2.) The Mood of Amotion rests on an immediate corollary of the principle, the same which is used for indirect demonstration. The inference expressed in the major premise being assumed to be valid, its consequent cannot be false, unless because its antecedent is so: if the consequent is false, the antecedent must be so likewise. Thus we gain the law of the mood, which is this: If the consequent is denied, there may be inferred the contradictory of the antecedent. The minor premise, therefore, is any proposition expressing an absolute denial of the consequent: the conclusion is any proposition expressing an absolute denial of the antecedent. In applying these rules to common terms, we have to bear in mind the rules of opposition. "If X is Y, X is Z; but X is not Z: therefore X is not Y." "If no X's are Y's, all X's are Z's; but some X's are not Z's: therefore some X's are Y's." "If some X's are Y's, some B's are Z's; but no B's are Z's: therefore no X's are Y's."

(3.) No conclusion can be drawn, with any minor premise, from either of the other two assumptions which are possible; the affirmation of the consequent, the denial of the antecedent. Neither of these assertions would affect the character of the major premise as an inference.

(4.) All hypothetical syllogisms may be reduced to categoricals. The positive mood falls directly into the first figure, the amotive mood into the second. Our second example in the first mood might easily be resolved, thus, into *Celarent;* "Any X's which are Y's are not any Z's; but all X's are some X's which are Y's: therefore no X's are any Z's." Often, however, the categorical expression

of the major premise becomes extremely unwieldy. In these cases the books, both English and foreign, advise the substitution of such unanalytic forms as those adopted in the following reduction, into *Camestres*, for the last example under the second mood: "All cases in which some X's are Y's, are some cases in which some B's are Z's; the present case is not any case in which some B's are Z's: therefore the present case is not any case in which some X's are Y's."

Analysis of the hypothetical syllogism.

110. Lastly, however, the frequency of such difficulties, in the reduction of hypotheticals, points significantly to inadequacy of the merely formal analysis. Other circumstances strengthen the suspicion.

The two syllogisms last treated exemplify an instructive distinction.

In the former of these, the conjunctive premise is, in effect, an *enthymeme;* and if for it there were substituted the missing premise, our given minor and conclusion would form with it a simple categorical syllogism: "No Y's are Z's; all X's are Y's: therefore no X's are Z's." But this is nothing more than explicating fully the conjunctive premise itself. There is no difference, logically appreciable, between the argument just set down, and this other: "If no Y's are Z's, and if all X's are Y's, no X's are Z's." Indeed, if we take up our ground very firmly, it may become probable to us that the latter form, as enouncing merely the relation of antecedent and consequent, is the precise and proper logical expression of the argument; that the former expresses too much, in respect that it seems to assert the truth of the antecedent, an assertion which does not logically come into question. In truth, as has already been shown,

categorical predication and inference rest on presuppositions. In actual thinking, we have, or endeavour to find, positive reasons, justifying the throwing of our thoughts into the categorical form. But logic is indifferent to these reasons: it accepts the assumptions as given; and it traces them to their consequences at the risk of those who give them.

But, again, the last of our examples exhibits another relation. The terms of the conjunctive premise are four; the inference which it alleges must be a complex inference, for the explication of which materials are not given. Some logicians are inclined to refuse to such propositions the character of genuine hypotheticals; but this denial rests on a narrow view of the process.[1] It is quite conceivable that we should make such an assertion as this: "If all good actions are self-rewarding, there are men who are independent of worldly honours;" and, if the relation between the antecedent and the consequent is admitted, we may argue from this premise, either by position or by amotion. But the relation must be admitted, if the argument is to have any force, or even any meaning; and the character of the data makes it impossible for us to verify the relation, unless we ampliate our reasoning by assuming premises which are not given, and which have terms wanting also in our data.

In both of our examples, however, presuppositions are

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[1] See Mansel, *Prolegomena Logica*, pp. 216, 217. Drobisch (p. 51) holds all hypothetical judgments, and also all disjunctives, to be truly synthetic; and his principle lies at the root of the view here taken of both.

absolutely required. The cases differ only in the amount and kind of these. In neither of them is the hypothetical proposition, as given, sufficient to determine the validity of the inference: it must be assisted, for both, by assumptions throwing us back on categorical forms and categorical laws.

All the doubts converge on one question, suggesting an answer that may be challenged as a paradox. Is there, in a categorico-hypothetical syllogism, any actual inference whatever? Is it really any thing more than the statement of a categorical inference, followed by an assertion (involving no inference at all), that there is an actual case on which that inference bears?

The structure and rules of the disjunctive syllogism.

111. As, in hypothetical propositions, the validity of the asserted inference must be assumed; so, in Disjunctives, it must be assumed, first, that the alternatives exclude each other; secondly, that the alternatives given are the only alternatives possible. If either of these assumptions is withheld, we are in the same position as we should be placed in by denying the validity of the hypothetical inference. No conclusion could be drawn from the disjunctive, whatever premise might be taken with it.

Categorico-Disjunctive Syllogisms (oftenest called simply Disjunctive) have two moods, which may be named, with explanation, like those which we have from hypotheticals. The mood in which the conclusion is affirmative may be called the Constructive, or the Mood of Position; that in which the conclusion is negative may be called the Destructive, or the Mood of Amotion. But the introduction of negation into the disjunctive premise, through the alternative, creates a contrariety of relation between the minor premise and the conclusion. When the conclusion is affirmative, the minor

premise must be negative; when the conclusion is negative, the minor premise must be affirmative.[1]

All alternatives but one being denied in the minor premise, the remaining alternative must be categorically affirmed in the conclusion: all alternatives but one being affirmed in the minor premise, the remaining alternative must be categorically denied in the conclusion. These assertions are, for the two moods, rules governing all cases in which the conclusion can be categorical. If the affirmations or denials of the minor premise were to fall short of exhausting all the alternatives but one, the conclusion must be disjunctive; and the argument would thus be taken out of the class of inferences here in question.

The rules are easily traceable to the law of non-contradiction, in its application to the simplest case that can yield real inference. Let there be given two names of thinkable objects, B and C. There is then possible the disjunctive assertion: "Either B is C, or B is not C." Since B cannot be both C and Not-C, it follows, first, that, if B is not Not-C, it must be C; next, that, if B is C, it is not Not-C. The introduction of a third object would give a positive term, as D, which must be thought as equivalent to Not-C, that is, as being contradictory of C. "B is either C or D: if B is not C, it must be D: if B is C, it cannot be D." When the given alternatives are more than two, the principle

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[1] Hence, as it has correctly been observed, the scholastic names of the disjunctive moods, "Modus ponens," and "Modus tollens," have not the same aptness as in their other application. The first is properly the "Modus tollendo-ponens; ab amotione ad positionem:" the second is the "Modus ponendo-tollens; a positione ad amotionem."

must still be strictly adhered to. Any one alternative, or group of alternatives, being affirmed or denied in the minor premise, all the others must be held as together constituting the contradictory of that alternative or group. All the varieties of combination which many alternatives make possible, must be treated in the same way. Thus, let the given major premise be this: "B is either C, or D, or E." For categorical conclusions, we must fix on some one alternative, and exclude from it all the rest. Affirmatively we may conclude "B is neither C nor D; therefore B is E:" negatively, "B is either C or D; therefore B is not E."

Analysis of the disjunctive syllogism.

112. The books do not attempt the reduction of disjunctive syllogisms into categoricals, unless by first reducing them to hypotheticals, through change of the disjunctive premise. The change to be made on it is dictated by the character of the other premise. Thus, for the first example just given, the major premise would pass into the hypothetical proposition, "If B is neither C nor D, it is E:" for the second example, it would take this shape: "If B is either C or D, it is not E."

This reduction, through hypotheticals, not only leads us back towards categorical forms, but likewise exhibits clearly the principle of the reasoning. It exposes, further, the character of the antecedent process through which, when the terms are common terms (the only case deserving minute inspection), the disjunctive proposition has come into existence. Our last proposition of the sort is equivalent to the assertion, that the class B is constituted by the sub-classes C, D, and E; that "all" the objects we call B are contained in three sub-classes, to which, severally and exclusively of each other, we give the names C, D, and E. What, in this

view, is the minor premise? More particularly, what is its subject? Its subject is, "Some or certain B's;" or, "the B's we are thinking of." If the premise is affirmative, it is a direct subalternate of the major premise. It affirms that the B's in question are contained in one of the sub-classes, as C; or, alternatively, that they are included either in one or another of them, as C or D. It follows, inevitably, in the conclusion, that the B's in question are excluded from one and all of the sub-classes co-ordinate to those in which those B's have been included. If, again, the minor premise is negative, the conclusion is gained on the same principle. The B's in question, being excluded from certain of the sub-classes, must be included in some one of the others; and, if they have been excluded from all the sub-classes but one, they must be included in that one.

In short, the disjunctive syllogism, like the hypothetical, sets forth, in its major premise, the result of an antecedent process; and it adds to this, in the minor premise, an assertion of fact. The hypothetical sets forth a pre-formed inference; the disjunctive sets forth a pre-formed ordination of terms, issuing in a logical division. But the latter goes further than the former: it does seem to be a real inference. It silently assumes a premise which is formulized in the expression of the law of excluded middle ("B is either C or Not-C, and cannot be both"); or in the rule of predication thence derived, that co-ordinate terms must be denied of each other. Thus, the ordination and division explicated in the major premise being presupposed, our last two examples are analyzable into these categorical forms: (1.) All B's which are neither C's nor D's are E's; certain B's are B's which are neither C's nor D's: therefore certain B's are E's (*Barbara* or *Darii*). (2.) B's which

are either C's or D's are not E's; certain B's are B's which are either C's or D's: therefore certain B's are not E's (*Celarent* or *Ferio*).

DIVISION II.—INFERENCE FROM PREMISES INVOLVING ULTRA-SYLLOGISTIC SUBSUMPTIONS.

The structure and rules of the categorical sorites.

113. The name Sorites is given to a complex argument, resolvable, by expression of steps implied, into a series of simple syllogisms, in which the conclusion of each but the last becomes a premise in the next following.[1] It is needless to examine any of its kinds, except that in which all the propositions are categorical.

The sorites may take, by two several arrangements of the propositions, either of two forms. The one is the Direct, common, or Aristotelian; the other the Reversed, or Go-

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[1] Sorites (from σωρὸς, a heap), cumulative argument. "Quemadmodum Soriti resistas? quem, si necesse sit, Latino verbo liceat *Acervalem* appellare: sed nihil opus est." (Cicero, *De Divinatione*, lib. ii., cap. 2.) The Germans call it the "chain-syllogism" (Kettenschluss). Most of them, also, give the name of "syllogistic-chain" (Schlusskette) to a form of argument which requires only a passing notice, that which the old logicians usually called the Epicheirema. It is a syllogism in which one or both of the premises are enthymemes; as this: "M is P (because M is C): S is M (because S is D): therefore S is P." The parenthetical assertions evidently exercise no influence on the conclusion: they are given only as reasons for admitting the premises. If a complete explication of the argument were required, we should have to construct two other syllogisms by supplying the missing premises, "C is P," and "D is M." These "prosyllogisms" being set down side by side, there might

clenian.[1] The rules of either are readily gained from those of the other. The first of the two may serve as our model: it is both the more commonly treated, and by much the more natural.

The following argument exemplifies the Direct Sorites: "A is M; M is N; N is P; P is Q; Q is B: therefore A is B." The last proposition is the only one presented as a conclusion: all the others appear as premises. The predicate of each premise, except the last, becomes the subject of the next premise: the conclusion has for its subject the subject of the first premise, for its predicate the predicate of the last.

The sorites is resolvable into a number of simple syllogisms, less by one than the number of its premises. Thus, our example, having five premises, yields four syllogisms. Of these, again, the sorites expresses no conclusion, except that of the very last, which becomes the conclusion of the sorites itself: it expresses no minor premise, except that of the first syllogism, which is the first proposition of the sorites: all the other premises are majors.

The extricated syllogisms of our example are the following; and in these it is observable, that the subject of the conclusion passes on as the subject of each minor premise.

1.	2.	3.	4.
M is N;	N is P;	P is Q;	Q is B;
A is M:	(A is N):	(A is P):	(A is Q):
(∴ A is N.)	(∴ A is P.)	(∴ A is Q.)	∴ A is B.

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be placed below them, as "episyllogism," the given syllogism, with its premises freed from their enthymematic supplement.

[1] From Goclen or Goclenius of Marburg, who, about the end of the sixteenth century, first analyzed it.

The rules of the sorites are readily deducible from this analysis.

(1.) All the constitutive syllogisms must be in the First Figure.[1] When the conclusion is negative, the second figure is reachable, but only through conversions; and when the conclusion is in A, all the indirect figures are plainly inapplicable.

(2.) Only one Premise can be Particular; and that must be the first of the expressed series. The reason is evident. All the others are major premises; and, in the first figure, the major must be universal.

(3.) Only one Premise can be Negative; and that must be the last of the expressed series. If any other were negative, the suppressed conclusion of its syllogism must be negative. But this conclusion becomes the suppressed minor premise of the next syllogism; and that premise must, in the first figure, be affirmative.

(4.) The Conclusion of the sorites may be an A, when all the premises are A: it may be an I, when the first premise is I, and all the others A: it may be an E, when the last premise is E, and all the others A: it may be an O, when the first premise is I, the last E, and all the others A.

The Reversed Sorites differs from the Direct in the order of the premises only, which is exactly transposed. The same example, so treated, stands thus: "Q is B; P is Q; N is P; M is N; A is M: therefore A is B." Here the subject of each premise but the last becomes the predicate of the next; the conclusion takes its subject from the last premise, its

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[1] See, however, as to this question, Lambert, *Neues Organon*, i., p. 188–190; Twesten, *Logik*, pp. 133, 138; Bachmann, *Logik*, p. 254; Drobisch, *Neue Darstellung*, pp. 116–124.

predicate from the first. The premises now expressed are the minors of the constitutive syllogisms, excepting the first premise, which is a major. The only changes which the rules of the common sorites undergo are these; that the premise which may be particular is the last, that which may be negative the first.

The extricated syllogisms are the following. The series is necessarily different from that yielded by the other form; and the predicate of the conclusion does duty as predicate of each major premise.

1.	2.	3.	4.
Q is B;	(P is B);	(N is B);	(M is B);
P is Q:	N is P:	M is N:	A is M:
(∴ P is B.)	(∴ N is B.)	(∴ M is B.)	∴ A is B.

Analysis of the categorical sorites.

114. The dissection of the sorites into simple syllogisms is not necessary. If it is accepted as given, the force of the reasoning is quite as evident as the dissection could make it; while the process may still more easily be referred to a higher principle.

Suppose (and the case is supposable, though not more), that, in an argument as complex as that in the example, all the terms are singulars. Each affirmation is then an assertion, that subject and predicate are but two names for one and the same individual object. Quantity not being in question, the quality of the propositions is the only point to be considered. Evidently the rule of the direct sorites holds: negation is admissible only at the last step. If it intruded earlier, the chain of identities would be broken; and any further assertions of identity would have no bearing on those that had preceded.

If the terms are common terms, the same principle is applicable, with this limitation only: that our affirmations are now assertions of inclusion, ("All A's are some M's"); while our negations are assertions of exclusion, "(The Q's are not any B's"). The antecedent of our thinking, the term whose relations are in question, is A, the subject of the conclusion. The common sorites (in this, as in most other points, an apter development of the argument than the other), deals with this term by a regular process of generalization. It begins by asserting the inclusion of A in the class M, that is, its identity with some of the M's: it next includes this class M in the wider class N; N in the wider class P; P in the wider class Q; and Q in the wider class B. Hence follows necessarily the inclusion of A in B, the widest of all the classes, that is, the identity of A with some of the B's.

The affirmative sorites does, in fact, nothing else than explicate, step by step, the affirmations implied in a series of terms positively pre-ordinated in extension thus, from highest to lowest: B, Q, P, N, M, A. Of each subordinate term, its superordinate may be affirmed universally: of the lowest of all the terms, A, the highest, B, may be so affirmed. The syllogisms, evolvable out of the common sorites, trace the A, stage by stage, from lower class to higher. But the case is parallel to that, already observed on, of the supposed suppression of the minor premise in a simple syllogism: the evidence which supports the reasoning is in as little need of the minute explication here as there.

The rules show themselves spontaneously, when the argument is regarded in the aspect just described. If our antecedent is "some A's," then of "some A's" only, through-

out the process, can either affirmation or denial take place: the first premise is particular; so must be the conclusion. It is equally manifest that no premise but the first can be particular. The inclusion of a term in a class would avail us nothing, unless we were able, in our next step, to include the whole of that class in the next higher. Again, if negation is introduced at any step before the last, the chain of the positive ordination has snapped. In asserting, for instance, not that "the N's are P's," but that "the N's are not P's," we should pass from the series of terms with which we began, A, M, N, into a new series, P, Q, B, of whose relations to the first series we know nothing. At our last step the crossing of the frontier is safe; because our journey is at an end. Instead of asserting that "the Q's are B's," we might assert that "the Q's are not B's;" whence it would follow that the A's, already identified with some of the Q's, are not B's.

The clue thus furnished would make the scrutiny of the Reversed Sorites very instructive. Its assertions proceed in the order of specification; but they necessarily oscillate. They must do so in order that,—while they began by asserting something of Q, the highest of the subordinate terms,—each lower term in its turn may, through a higher, be directly connected with the superordinate B, till the lowest specification is reached, and A brought into relation with B.

The evolved syllogisms of the two forms bring up curiously, too, the bearings of the two wholes of the concept. The direct sorites is evolved through repeated dealing with A, as an object or objects to be referred to classes till it reaches B. The reversed sorites is evolved through repeated dealing with B, as an attribute to be predicated of object after object, till at last it becomes possible to

predicate it of A. The former proceeds in extension, the latter in comprehension.

DIVISION III.—INFERENCE BY COMBINATION OF COMPLEX MODES.

The mixed sorites and the dilemma.

115. The complex forms of predication and reasoning which have now been examined, admit various combinations, which have been, by many logicians, scrutinized with great patience and sagacity. But the theory of them cannot be said to be perfect; and they are certainly curious rather than useful. All of them carry us, by a greater or less distance, still further away from that direct comparison of terms, which, as expressed in categorical propositions, we have had to accept as the normal form of explicative thinking.

It must here suffice to point out, very generally, some of the most prominent among those complexly complicated shapes of reasoning.

I. A Sorites may be constructed with propositions all of which are Hypothetical. Or all its premises may be Hypothetical, except the last: this premise being Categorical, so will be the conclusion.—To the Disjunctive Sorites, almost all logicians have refused admission; and rightly. It is quite possible; but, yielding nothing except a growing congeries of alternatives, it expresses only a deeper and deeper plunging into doubts.

II. The very complex argument, called the Dilemma, has a celebrity which claims for it somewhat closer attention. When expressed so as to bring out all its elements, it is describable as being an Hypothetico-Disjunctive Syllogism. Its major premise is an hypothetical proposition, one of whose constitutive propositions (either antecedent or con-

sequent) is categorical, the other disjunctive. The minor premise is in form disjunctive, and may be either affirmative or negative. (1.) The minor premise may affirm, exhaustively, the disjunctive proposition of the major; and, in this case, the conclusion affirms the categorical proposition of the major. But this inference is valid only when the major premise has its categorical proposition as consequent. (2.) The minor premise may deny, exhaustively, the disjunctive proposition of the major; and in this case the conclusion denies the categorical proposition of the major. This inference is valid only when the major premise has its categorical proposition as antecedent. In short, there are thus two moods, corresponding in character to the constructive and destructive moods in hypotheticals. The argument may be analyzed and tested as an hypothetical.

The following are examples.—1. (*Major*) If either A is B, or E is F, then C is D; (*Minor*) Either A is B, or E is F: (*Conclusion*) Therefore C is D.—2. If A is B, then either C is D, or E is F; but neither C is D, nor E is F: therefore A is not B.[1]

The Greek dialecticians prided themselves on the exhibition of dilemmas which they alleged to be insoluble. These

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[1] Thomson, *Laws of Thought*, p. 267; Fries, *System*, p. 61. The name of Dilemma is by some logicians used more widely than here: by others it is perversely limited to the sophistical arguments spoken of in the next paragraph. The name was most probably applied to this kind of inference, to intimate the compound character of the disjunctive assumption (λῆμμα). The argument was also called by the Latins the "syllogismus cornutus;" whence the phrase of "placing one on, or between, the horns of a dilemma." The word Dilemma supposes two alternatives only: if the alternatives are more than two, the argument is properly a Trilemma

were arguments so framed, that it is necessary to admit both the affirmative minor premise and the negative, and thus to reach both of two contradictory conclusions. All such arguments must, of course, have a fallacy somewhere. Several of the ancient examples are constructed so dexterously, that the detection of the flaw is difficult; but it is always possible, while often there are more flaws than one.—In the first place, the arguments are sometimes not given in the form just described; and, when their propositions are examined, it is found that they cannot be thrown into that form, or into any other that guarantees any conclusion. In such cases, the fallacy is formal, and logically discoverable. Next, if a genuine form is given or attainable, the admission of the conclusion, as a logical consequent of the premises, leaves the argument worthless, unless there have concurred three conditions, all material or extralogical. (1.) The disjunctive proposition of the major must be a genuine disjunctive: its alternatives must be both exclusive and exhaustive. Here, more probably than elsewhere, will be found the weakness of a sophistical dilemma: either it ignores some alternative thinkable under the terms; or it asserts, as mutually exclusive, cases which are reconcileable. (2.) The inference, hypothetically stated in the major,

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or Polylemma. There are three alternatives in the first of the following examples. "A chess-player may argue thus: Whether I move my king, or cover him, or take the piece which has given him check, I must be checkmated at the next move; but I must do one or another of the three things: therefore I must be checkmated at the next move." (Drobisch, p. 111.) "If man is incapable of improvement, he must be either a divinity or a brute; but man is neither the one nor the other: therefore man is not incapable of improvement." (Troxler, ii. 103.)

must be valid, either *ex facie* or through extraneous suppositions. (3.) The assertion of fact made in the minor premise must be admitted as true.[1]

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[1] It may be worth while to illustrate, by two of the most famous among the ancient examples, the complications through which it was attempted to veil the weak points of sophistical dilemmas.

The first is known as the "Syllogismus Crocodilinus."—A crocodile, having seized an infant, promises to give it back if the mother will say truly what is to happen to it. She, perhaps rashly, asserts, "You will not give it back." Thereupon both parties play the sophist. The crocodile argues thus: "If you have spoken truly, I cannot give back the child without contradicting your assertion; if you have spoken falsely, I cannot give it back, because you have not fulfilled the agreement: therefore I cannot give it back, whether you speak truly or falsely." The mother replies: "If I have spoken truly, you must give back the child in terms of the agreement; if I speak falsely, this can only be because you have given back the child: therefore, in either view, the child must be given back."

The other example is the "Sophism of Euathlus," which might have been named, quite as fairly, from the other party to the dispute. Neither of them is represented as having been more successful than the crocodile or the mother, in discovering, for the division on which the disjunctive rests, a foundation justly applicable to the facts of the case.—Euathlus had received lessons from Protagoras, the rhetorician, on condition that the fee should be paid if the pupil were successful in the first cause he pleaded. Euathlus delaying to undertake any cause, Protagoras sues him; and this is consequently the young man's first law-suit. The master argues in this way: "If I am successful in the cause, you must pay me in virtue of the sentence; if I am unsuccessful, you must pay me in fulfilment of the contract." The pupil retorts: "If I am successful, I am free by the sentence; if I am unsuccessful, I am free by the contract."

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